3rd Order PLL Design

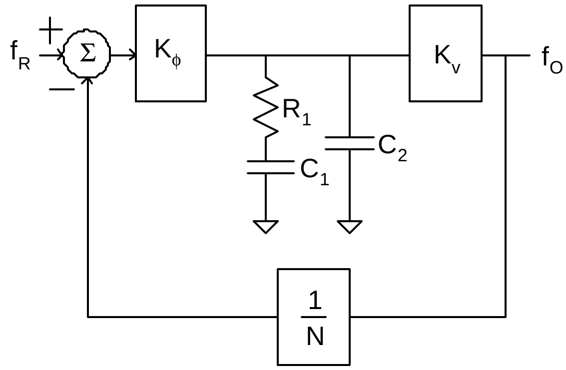

Fig. 1: 3rd-Order PLL with Current-Mode Phase-Detector

- useful functions and identities
- Units
- Constants

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Introduction

The charge-pump-based third order PLL is the most popular PLL architecture today. This report discusses the design and analysis and the loop dynamics of 3rd order PLLs. This includes phase margin sizing to minimize the settling time of the PLL, and a anaylsis of a modified loop filter. There are other PLL topologies, such as those which boost the current charge pump current initially for faster settling, these will be discussed in a later report. There are also op-amp based loop filter topologies as shown in the following figure.

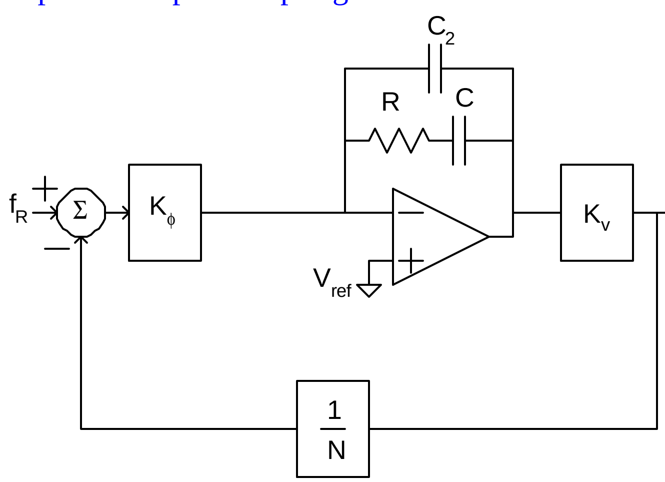


Fig. 3: 3rd-Order PLL w/ Current-Mode Phase Detector and Extended Charge-Pump Output Swing w/Op-Amp.

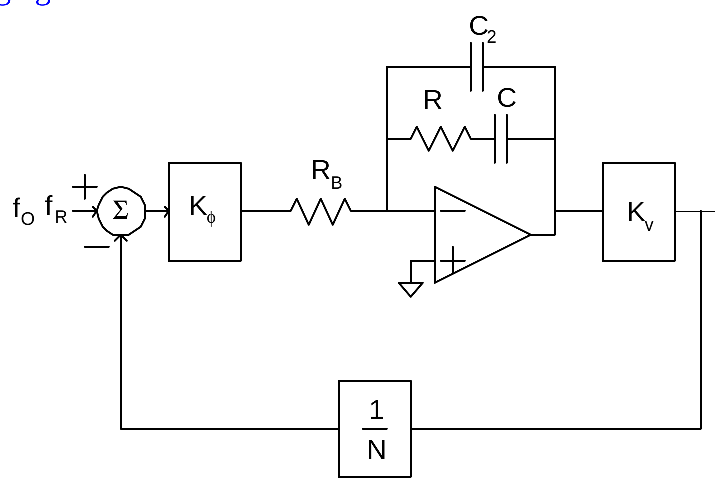


Fig. 3: 3rd-Order PLL with Voltage-Mode Phase-Detector

Opamp-based loop filter topologies have the advantage of a larger control voltage range for the VCO and better UP/DN current matching. They have the disadvantage of higher noise and thus more current. The passive component sizing is the same for opamp-based PLL as for the passive only. Opamp PLLs will be discussed in a later report as well.

Inputs

$f_{step} := 25\text{MHz}$ — Maximum output frequency step

$f_o := 1\text{GHz}$ — Output Frequency

$f_r := 1\text{MHz}$ — Reference frequency

$f_{acc} := 1\text{kHz}$ — Acceptable frequency error for settling

$PM_{des} := 60\text{deg}$ — Desired phase margin

$I := 20\mu\text{A}$ — Charge pump current

$K_v := 2\cdot\pi\cdot 15\cdot\dfrac{\text{Mrad}}{\text{sec}\cdot\text{volt}}$ — VCO Gain

$num := 100$ $\quad i := 1..num$ — Number of points for plotting

$f_u := 10\text{kHz}$ $\quad \omega_u := 2\cdot\pi\cdot f_u$ — Desired unity gain bandwidth

Preliminary Calculations

$N := \dfrac{f_o}{f_r}$ $\quad N = 1\times 10^3$ — Divider ratio

$K_\phi := \dfrac{I}{2\cdot\pi}$ — Tri-state charge pump gain

3^{rd} Order PLL Design Procedure

Without a loop filter PLLs are a type I system a single pole at DC from the VCO voltage to phase transfer function. The loop filter accumulates the average charge from the charge pump to generate a fixed voltage to set the frequency of the VCO. This accumulation gives the loop another pole at DC. Thus the loop starts with a phase of -180 degrees. In order to insure stability and provide a smooth settling response the phase must be raised with a zero in the loop to provide a desired phase margin at the unity gain frequency of the open loop transfer function. This zero is introduced by adding a resistor in series with the main loop filter capacitor.

To size the loop filter to provide a desired phase margin we first start by finding the open-loop unity-gain bandwidth. This requires the open-loop transfer function of the loop as shown in the following equation.

$$GH(s) = K_\phi \cdot \frac{1}{(C_1 + C_2)\cdot s} \cdot \frac{R_1 \cdot C_1 \cdot s + 1}{1 + R_1 \cdot \frac{C_1 \cdot C_2}{C_1 + C_2}\cdot s} \cdot \frac{K_v}{s} \cdot \frac{1}{N} = \frac{K_\phi \cdot K_v}{N\cdot (C_1 + C_2)\cdot s^2} \cdot \frac{\frac{s}{\omega_z} + 1}{1 + \frac{s}{\omega_p}}$$

One design methodology is to place the zero below the unity gain bandwidth to increase the phase margin. The pole is placed an equal factor above the unity gain bandwidth. Let's call this factor wu_wz, then the transfer function becomes:

$$\omega_p = \omega_u \cdot wz\_wu \qquad \omega_z = \frac{\omega_u}{wz\_wu} \qquad GH(s) = \frac{K_\phi \cdot K_v}{N\cdot (C_1 + C_2)\cdot s^2} \cdot \frac{wu\_wz \cdot \frac{s}{\omega_u} + 1}{1 + \frac{s}{\omega_u \cdot wu\_wz}}$$

The magnitude of this transfer function at ω_u is given by the following equation

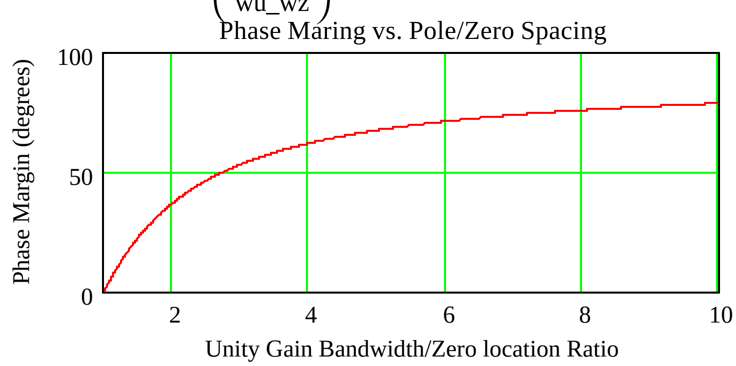
$$MagGH(\omega_u) = \frac{K_\phi \cdot K_v}{N\cdot (C_1 + C_2)\cdot \omega_u^2} \cdot wu\_wz$$

Now by setting the loop gain equal to unity, the unity gain bandwidth can be solved for with the result given in the following equation:

$$\omega_u = \sqrt{\frac{wu\_wz \cdot K_\phi \cdot K_v}{N \cdot (C_1 + C_2)}}$$

We can now use the equation for the unity gain bandwidth to solve for the phase margin, which is plotted below the equation as a function of the unity-gain bandwidth to zero ratio.

$$PM(wu\_wz) := atan(wu\_wz) - atan\left(\frac{1}{wu\_wz}\right)$$



If phase margin is given, we can solve for the variable wu_wz. There is not closed form solution to this expression, so we use a root finder. An initial guess of 4 is used.

$$\omega u\_\omega z(PM) := \left| \begin{array}{l} wu\_wz \leftarrow 4 \\ root\left[\left(atan(wu\_wz) - atan\left(\frac{1}{wu\_wz}\right)\right)\cdot \frac{180 \cdot deg}{\pi \cdot rad} - PM, wu\_wz\right] \end{array} \right.$$

$wu\_wz := \omega u\_\omega z(PM_{des})$ $\quad wu\_wz = 3.732$

Optimal Phase Margin for Minimum Settling

Part of the design process involves entering a desired value of phase margin. The phase margin can effect the performance of several parameters including phase noise, loop bandwidth, and settling time. It's effect on some parameters is minor for typical values. Phase margin has it's biggest impact on settling time. Thus, settling time is used to optimized phase margin. The variable ωu_ωz is easier to work with than phase margin, so it will be used. There is a direct relationship between the two, which will be used to find the optimum phase margin.

$$\mathrm{ClosedLoop}(s) = \frac{\left(wu\_wz\cdot\frac{s}{\omega_u}+1\right)\cdot N}{wu\_wz\cdot\left(\frac{s}{\omega_u}\right)^2+\left(\frac{s}{\omega_u}\right)^3+wu\_wz\cdot\frac{s}{\omega_u}+1}$$

The transient response of this system is:

$$h(\omega ut, wu\_wz) := f_{step}\cdot\left[1+\frac{2\cdot e^{\left[-\left(\frac{wu\_wz-1}{2}\right)\cdot\omega ut\right]}\cdot\cosh\left(\frac{1}{2}\cdot\sqrt{wu\_wz+1}\cdot\sqrt{wu\_wz-3}\cdot\omega ut\right)-e^{-\omega ut}\cdot(wu\_wz-1)}{wu\_wz-3}\right]$$

For the case where wu_wz=3 the response is

$$h(\omega ut, wu\_wz) := f_{step}\cdot\left[1+\left(\omega ut^2-\omega ut-1\right)\cdot e^{-\omega ut}\right]$$

We can write a single expression for the transient response in both cases, when wu_wz is and isn't 3.

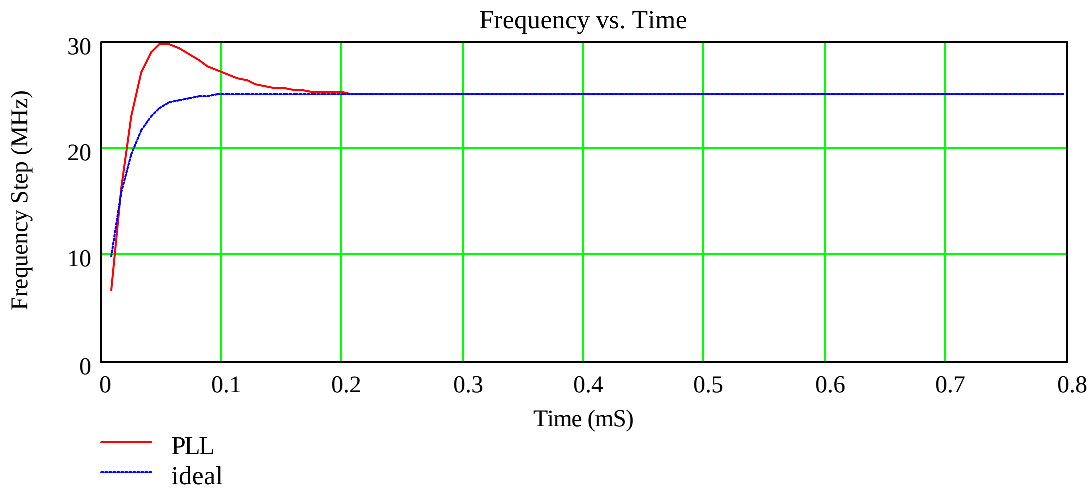
$$h_{lin}(\omega ut,\beta) := \mathrm{if}\left[\beta = 3, f_{step}\cdot\left[1+\left(\omega ut^2-\omega ut-1\right)\cdot e^{-\omega ut}\right], f_{step}\cdot\left[1+\frac{2\cdot e^{\left[-\left(\frac{\beta-1}{2}\right)\cdot\omega ut\right]}\cdot\cosh\left(\frac{1}{2}\cdot\sqrt{\beta+1}\cdot\sqrt{\beta-3}\cdot\omega ut\right)-e}{\beta-3}\right.\right.$$

It is interesting to compare the settling to an ideal linear settling response with a bandwidth of wu.

$$h_{ideal}(\omega ut) := f_{step}\cdot\left(1-e^{-\omega ut}\right)$$

A plot of the transient response are given below

$$t_i := \frac{i}{num}\cdot\frac{50}{\omega_u}$$



We can also find the frequency error response

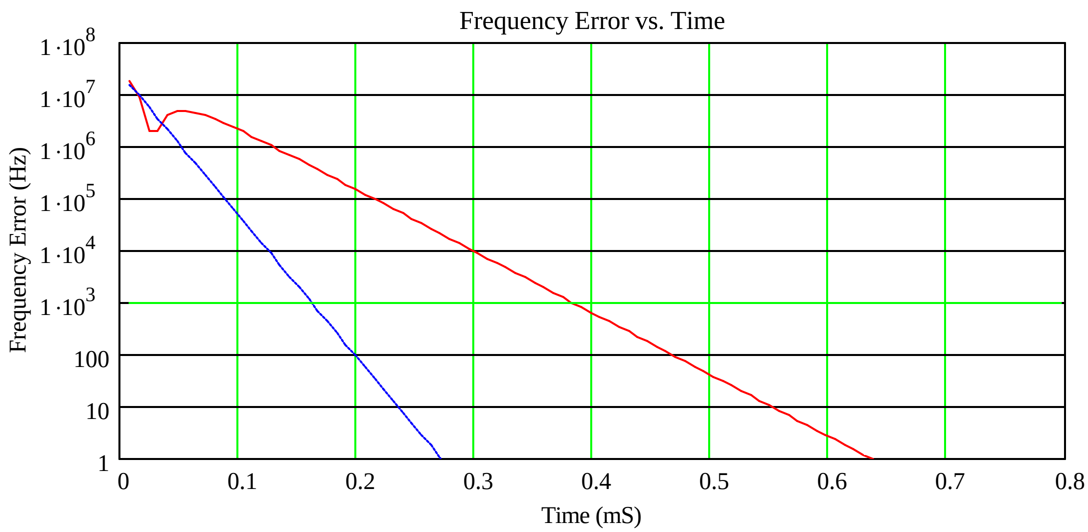
$$f_{err}(t,\beta) := f_{step}\cdot \text{if}\left[\beta = 3, \left[(\omega_u\cdot t)^2 - \omega_u\cdot t - 1\right]\cdot e^{-\omega_u\cdot t}, \frac{2\cdot e^{-\left(\frac{\beta-1}{2}\right)\cdot\omega_u\cdot t}\cdot\cosh\left(\frac{1}{2}\cdot\sqrt{\beta+1}\cdot\sqrt{\beta-3}\cdot\omega_u\cdot t\right) - e^{-\omega_u\cdot t}\cdot(\beta-1)}{3-\beta}\right]$$

$f_{err}(3mS, 3) = 0\,Hz$

And compare it to an ideal error response

$$f_{errideal}(t, wu\_wz) := f_{step}\cdot e^{-\omega_u\cdot t}$$

Note that it is best to plot the error response on a logarithmic scale vs. time, because the loop settles exponentially.



The following function calculates the settling time of the transient response by looking for the last time the error crosses the acceptable error limit.

$$t_{settle}(wu\_wz) := \begin{array}{|l} t_{val} \leftarrow 1sec \\ \text{for } i \in 2..num \\ \quad t_{val} \leftarrow \text{if}\left[\left(\left|f_{err}(t_i, wu\_wz)\right| \le f_{acc}\right)\cdot\left(\left|f_{err}(t_{i-1}, wu\_wz)\right| > f_{acc}\right), t_i, t_{val}\right] \\ t_{val} \end{array}$$

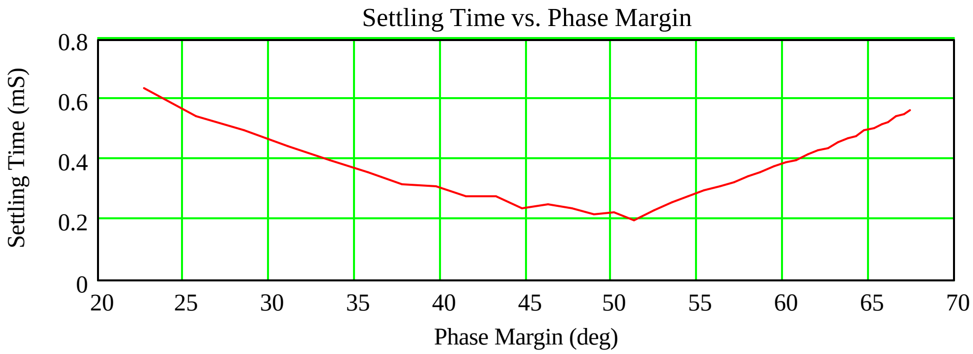
$t_{settle}(wu\_wz) = 0.382\,mS$

Now to visually find the optimum phase margin, we plot the settling time vs. phase margin. This is also useful for adding margin due to process variations

$numB := 40$ $\beta_{min} := 1.5$ $\beta_{max} := 5$

$i := 1 .. numB$

$$\beta_i := \frac{i-1}{numB-1} \cdot \left(\beta_{max} - \beta_{min}\right) + \beta_{min}$$



The following function sweeps through values of wu_wz to find the optimal value.

$$
wu\_wz_{opt} := \left|
\begin{array}{l}
t_{setmin} \leftarrow 10^{9} sec \\
\beta_{opt} \leftarrow 0 \\
\text{for } i \in 1 .. numB \\
\quad \left|
\begin{array}{l}
\beta \leftarrow \dfrac{i-1}{numB-1} \cdot \left(\beta_{max} - \beta_{min}\right) + \beta_{min} \\
t_{settle} \leftarrow \left|
\begin{array}{l}
t_{val} \leftarrow 10^{9} sec \\
\text{for } i \in 2 .. num \\
\quad t_{val} \leftarrow \text{if}\left[\left(\left|f_{err}\left(t_i, \beta\right)\right| \le f_{acc}\right) \cdot \left(\left|f_{err}\left(t_{i-1}, \beta\right)\right| > f_{acc}\right), t_i, t_{val}\right] \\
t_{val}
\end{array}
\right. \\
\beta_{opt} \leftarrow \text{if}\left(t_{settle} < t_{setmin}, \beta, \beta_{opt}\right) \\
t_{setmin} \leftarrow \text{if}\left(t_{settle} < t_{setmin}, t_{settle}, t_{setmin}\right)
\end{array}
\right. \\
\beta_{opt}
\end{array}
\right.
$$

$wu\_wz_{opt} = 2.846$

This corresponds to a phase margin of

$PM_{opt} := PM\left(wu\_wz_{opt}\right)$

$PM_{opt} = 51.282\, deg$

We increase this phase margin slightly to account for loop gain variations. Note that this value will change when slewing effects are added.

Synthesis of Loop Filter Components

With given values for ω_u, K_ϕ, K_v, and N and now wu_wz, we can solve for the loop filter components. The equations which must be solved simultaneously are given below:

$$\omega_u = \sqrt{\frac{wu\_wz \cdot K_\phi \cdot K_v}{N \cdot (C_1 + C_2)}}$$

$$\omega_u = \frac{C_1 + C_2}{R_1 \cdot C_1 \cdot C_2} \cdot \frac{1}{wu\_wz}$$

$$\omega_u = \frac{wu\_wz}{R_1 \cdot C_1}$$

The solution to these three equations are the following loop filter components

$$C_2 := \frac{K_\phi \cdot K_v}{\omega_u^{\,2} \cdot N \cdot wu\_wz} \qquad C_2 = 0.02\,\mathrm{nF}$$

$$C_1 := \frac{K_\phi \cdot K_v \cdot \left(wu\_wz^2 - 1\right)}{\omega_u^{\,2} \cdot N \cdot wu\_wz} \qquad C_1 = 0.263\,\mathrm{nF}$$

$$R_1 := \frac{\omega_u \cdot N \cdot wu\_wz^2}{K_\phi \cdot K_v \cdot \left(wu\_wz^2 - 1\right)} \qquad R_1 = 225.64\,\mathrm{k\Omega}$$

If these equations are plugged back into the original transfer function, the expression simplifies to

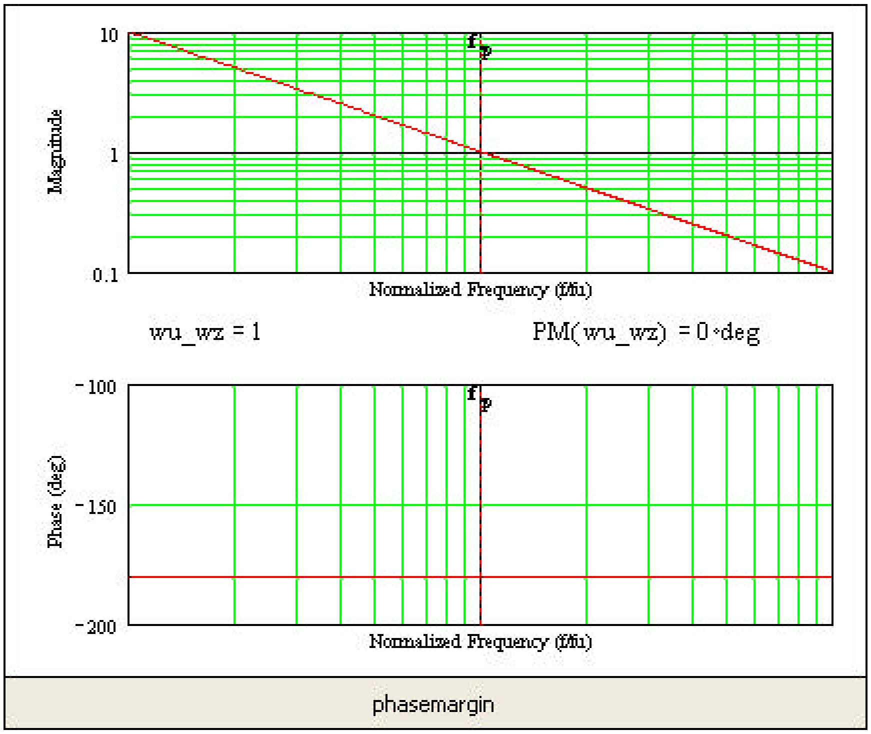
$$GH(s) := \frac{1}{wu\_wz \cdot \left(\frac{s}{\omega_u}\right)^2} \cdot \frac{wu\_wz \cdot \frac{s}{\omega_u} + 1}{1 + \frac{s}{\omega_u \cdot wu\_wz}}$$

We can use this to plot the magnitude and phase response for different values of wu_wz as shown in the following plots normalized to 1Hz.

$$AngleGH(f\_fu) := -180deg + atan(wu\_wz \cdot f\_fu) - atan\left(\frac{f\_fu}{wu\_wz}\right)$$

$$MagGH(f\_fu) := \frac{1}{wu\_wz \cdot (f\_fu)^2} \cdot \sqrt{\frac{(wu\_wz \cdot f\_fu)^2 + 1}{1 + \left(\frac{f\_fu}{wu\_wz}\right)^2}}$$

Here is a little movie of the gain and phase plots changing with wu_wz



The closed loop transfer function for the loop is

$$fo\_fi(s) = \frac{\left(wu\_wz\cdot\frac{s}{\omega_u} + 1\right)\cdot N}{wu\_wz\cdot\left(\frac{s}{\omega_u}\right)^2 + \left(\frac{s}{\omega_u}\right)^3 + wu\_wz\cdot\frac{s}{\omega_u} + 1}$$

To determine how much of the reference spur gets filtered by the loop it is useful to find the transfer function for frequencies much higher than the loop bandwidth. Note that a 3rd order PLL has a second order roll-off, because of the zero used for stabilization.

$$10^{\frac{Spur_{atten}}{20}} = \left|fo\_fi\left(f_r\right)\right| = \frac{wu\_wz\cdot N}{\left(\frac{f_r}{f_u}\right)^2}$$

From this we can see it is desirable to make the wu_wz value smaller to reduce reference spurs, which corresponds to smaller phase margins.

Integrated Loop Filters

For some applications, the loop filter components are small enough to be integrated. The resistor area is usually negligible, but the capacitor area can be huge. For processes without special high-density capacitors, the highest density integration involves using MOS capacitors and resistors for the loop filter. The following equation illustrates the effect of certain loop variables on capacitor area.

$$Area_{cap} = \frac{K_{\phi} \cdot K_V \cdot wu\_wz^2}{(2 \cdot \pi \cdot f_r)^2 \cdot 10^{\frac{Spur_{atten}}{20}} \cdot C_{OX}} \qquad \text{where} \qquad f_u = f_r \cdot \sqrt{\frac{10^{\frac{Spur_{atten}}{20}}}{wu\_wz \cdot N}}$$

Without going into detail (as will be done in a later report), the area of the capacitor equation will become the following expression, when the effects of resistor noise are considered for sizing the charge pump current:

$$Area_{cap} = \frac{k \cdot Temp \cdot f_o^{1.5} \cdot 2 \cdot K_V^2 \cdot Hz \cdot wu\_wz^{2.5}}{(2 \cdot \pi)^3 \cdot f_r^{4.5} \cdot 10^{\frac{Spur_{atten} \cdot 1.5}{20}} \cdot C_{OX} \cdot 10^{\frac{-L_{filt\omega u}}{10}}}$$

From this equation we can see the capacitor area can be strongly reduced by reducing K_v (switched-capacitor VCO, low temperature sensitivity VCO), increasing reference frequency (fractional N, and operating VCO at multiples of desired frequency, and dividing down after PLL), lower phase margin (careful with settling!), lower spur attenuation (low spurious charge pump design), and higher budgeting of PLL phase noise at ωu to the resistor. This is usually possible, because the ωu is usually lower than the toughest phase noise requirement frequency.

Modified Loop Filter

Now lets find the component values for a slightly modified version of the loop filter. Here the deglitching capacitor, C_2, is placed across the main loop filter resistor instead of from the control line to ground. The transfer function is given in the following equation.

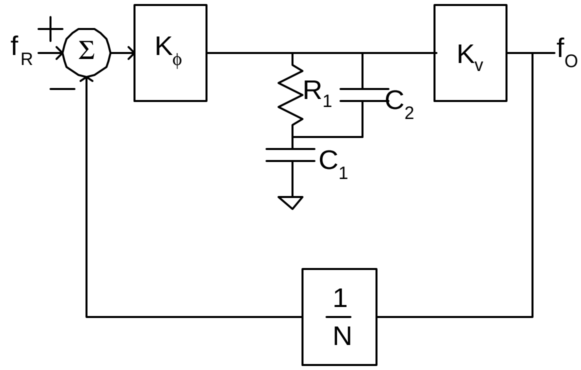


Fig. 1: 3rd-Order PLL with Current-Mode Phase-Detector and Modified Loop Filter (Johns and Martin)

$$GH(s) = K_\phi \cdot \frac{R_1 \cdot (C_1 + C_2) \cdot s + 1}{C_1 \cdot s \cdot (R_1 \cdot C_2 \cdot s + 1)} \cdot \frac{K_v}{s} \cdot \frac{1}{N} = \frac{K_\phi \cdot K_v}{N \cdot C_1 \cdot s^2} \cdot \frac{1 + \dfrac{s}{\omega_z}}{1 + \dfrac{s}{\omega_p}} = \frac{K_\phi \cdot K_v}{N \cdot C_1 \cdot s^2} \cdot \frac{wu\_wz \cdot \dfrac{s}{\omega_u} + 1}{1 + \dfrac{s}{\omega_u \cdot wu\_wz}}$$

The open-loop unity-gain bandwidth and phase margin of the circuit are:

$$\omega_u = \sqrt{\frac{wu\_wz \cdot K_\phi \cdot K_v}{N \cdot C_1}}$$

$$PM(wu\_wz) := \operatorname{atan}(wu\_wz) - \operatorname{atan}\left(\frac{1}{wu\_wz}\right)$$

Now the three expressions to solve for the loop filter components are

$$\omega_u = \sqrt{\frac{wu\_wz \cdot K_\phi \cdot K_v}{N \cdot C_1}}$$

$$\omega_u = \frac{wu\_wz}{R_1 \cdot (C_1 + C_2)}$$

$$\omega_u = \frac{1}{wu\_wz \cdot R_1 \cdot C_2}$$

The solution to these three equations are the following loop filter components:

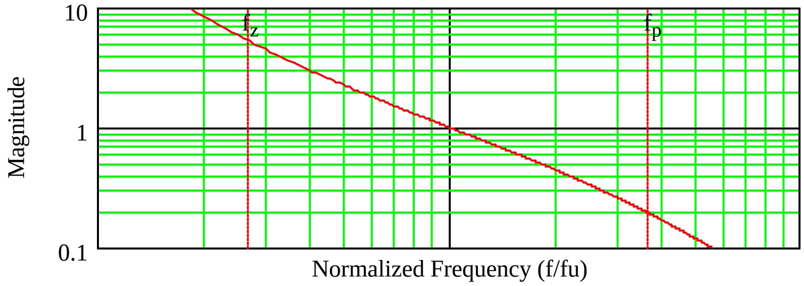
$$C_{1M} := \frac{wu\_wz \cdot K_\phi \cdot K_v}{\omega_u^{\,2} \cdot N} \qquad C_{1M} = 0.284\,\mathrm{nF}$$

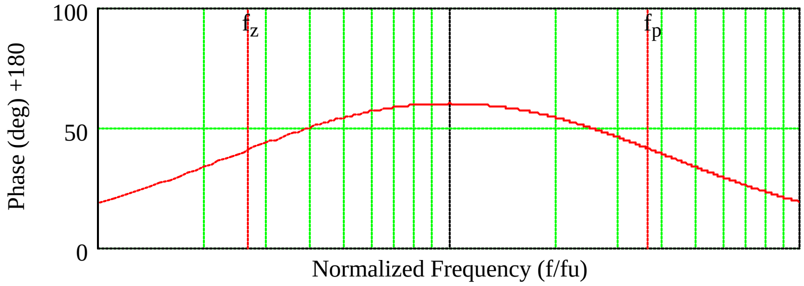
$$C_{2M} := \frac{wu\_wz \cdot K_\phi \cdot K_v}{\left(wu\_wz^2 - 1\right) \cdot \omega_u^{\,2} \cdot N} \qquad C_{2M} = 0.022\,\mathrm{nF}$$

$$R_{1M} := \frac{\left(wu\_wz^2 - 1\right) \cdot \omega_u \cdot N}{wu\_wz^2 \cdot K_\phi \cdot K_v} \qquad R_{1M} = 1.944 \times 10^5\,\Omega$$

We can check the frequency response, but it should be identical to the standard loop filter.

$$GH(s) := K_{\phi} \cdot \frac{R_{1M} \cdot (C_{1M} + C_{2M}) \cdot s + 1}{C_{1M} \cdot s \cdot (R_{1M} \cdot C_{2M} \cdot s + 1)} \cdot \frac{K_v}{s} \cdot \frac{1}{N}$$





For typical values we see that the modified loop filter has about 10% more capacitance, and thus area. The original structure is allows the current return paths to be better contained as the layout of a grounded capacitor is easier. Thus the original structure is preferred to the modified structure.

PLL Design Function

The entire design procedure is placed into the following function below, which is more useful for iterative calculations.

$$\mathrm{pll3rd}(f_r, f_o, I, f_u, K_V, PM) := \left| \begin{array}{l} x \leftarrow 4 \\ \beta \leftarrow \mathrm{root}\left[\left(\mathrm{atan}(x) - \mathrm{atan}\left(\frac{1}{x}\right)\right) - PM, x\right] \\ N \leftarrow \dfrac{f_o}{f_r} \\ K_\phi \leftarrow \dfrac{I}{2 \cdot \pi} \\ \omega_u \leftarrow 2 \cdot \pi \mathrm{rad} \cdot f_u \\ C_2 \leftarrow \dfrac{K_\phi \cdot K_V}{\beta \cdot N \cdot \omega_u^{\ 2}} \\ C_1 \leftarrow \dfrac{\left(\beta^2 - 1\right) \cdot K_\phi \cdot K_V}{\omega_u^{\ 2} \cdot N \cdot \beta} \\ R_1 \leftarrow \dfrac{\beta}{\omega_u \cdot C_1} \\ \begin{pmatrix} \dfrac{C_1}{\mathrm{farad}} \\ \dfrac{C_2}{\mathrm{farad}} \\ \dfrac{R_1}{\mathrm{ohm}} \end{pmatrix} \end{array} \right.$$

Here the pll3rd function is ran on some example parameters. Note the outputs are unitless, because Mathcad requires all the untis of a matrix to be the same.

$x := \mathrm{pll3rd}(f_r, f_o, I, f_u, K_V, PM_{des})$

$C_1 := x_1 \cdot \mathrm{farad}$	$C_1 = 0.263\,\mathrm{nF}$	Main Loop Filter Capacitor
$C_2 := x_2 \cdot \mathrm{farad}$	$C_2 = 0.02\,\mathrm{nF}$	Secondary Loop Filter Capacitor
$R_1 := x_3 \cdot \mathrm{ohm}$	$R_1 = 225.64\,\mathrm{k\Omega}$	Main Loop Resistor