2^{nd} Order PLL Design and Analysis

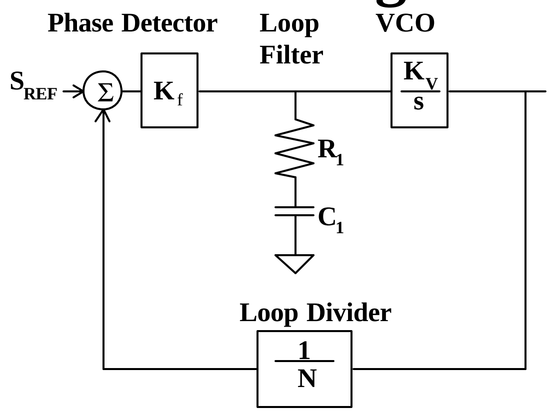

Fig. 1: 2^{nd} Order PLL with Current-Mode Phase Detector

▸ useful functions and identities
▸ Units
▸ Constants

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Introduction

Although the emphasis of PLL design is on 3rd-order PLLs, the settling time of second order PLLs is important, especially for boosted PLLs. In boost mode the transfer function of a third-order PLL sometimes reverts to a second order expression. As with 3rd-order PLL's there are different implementations, including voltage mode and those with operational-amplifier based phase detectors. This report will talk about the most popular, which uses a current-mode phase detector and a passive loop filter.

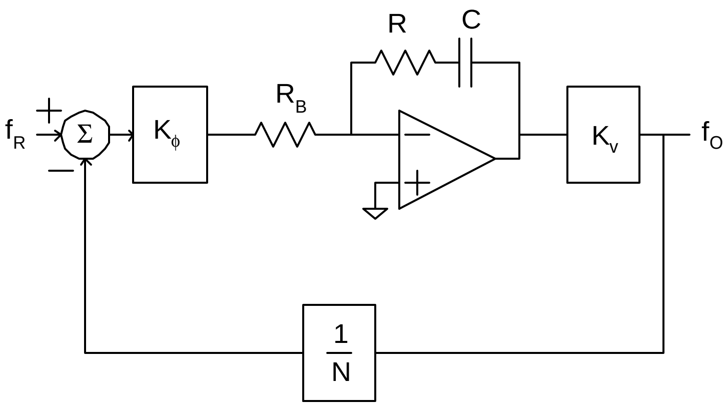


Fig. 2: 2nd-order PLL with voltage-mode phase detector and active filter

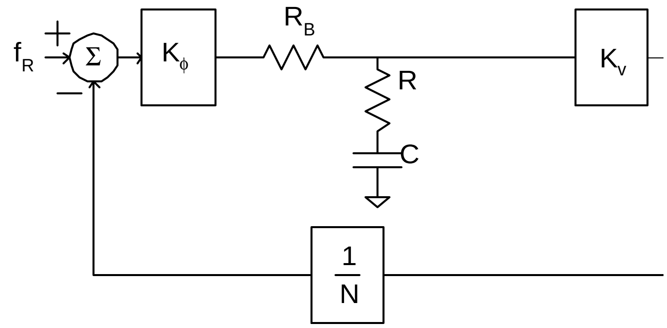


Fig. 3: 2nd-order PLL with voltage-mode phase detector and passive filter

Inputs

$f_r := 10 \cdot \text{MHz}$	Reference oscillator frequency and period
$f_o := 1 \cdot \text{GHz}$	Output frequency
$f_{acc} := 1\text{kHz}$	Acceptable Frequency Error
$f_{step} := 25\text{MHz}$	Maximum Frequency Step (output referred)
$I := 50\mu\text{A}$	Charge Pump Current
$f_u := 1 \cdot \text{kHz}$	Unity Gain Frequency
$K_v := 2 \cdot \pi \cdot 10 \cdot 10^6 \cdot \frac{\text{rad}}{\text{sec} \cdot \text{volt}}$	VCO Gain
$PM_{des} := 70\text{deg}$	Phase Margin (70 degrees recommended, 50 degrees if slewing is considered)

Initial Calculations

$T := \frac{1}{f_r}$	$T = 0.1\,\mu\text{S}$	Reference Period
$\omega_u := 2 \cdot \pi \cdot f_u$	$\omega_u = 6.283 \times 10^3 \,\frac{\text{rad}}{\text{sec}}$	Unity Gain Frequency
$N := \frac{f_o}{f_r}$	$N = 100$	Loop divider ratio
$K_\phi := \frac{I}{2 \cdot \pi}$	$K_\phi = 7.958 \,\frac{\mu\text{A}}{\text{rad}}$	Phase Detector Gain

Loop Filter Design Procedure

The open-loop transfer functions for the PLL are:

$$GH(s) = K_\phi \cdot \frac{R_1 \cdot C_1 \cdot s + 1}{C_1 \cdot s} \cdot \frac{K_v}{s} \cdot \frac{1}{N} = K_\phi \cdot \frac{\omega_z \cdot s + 1}{C_1 \cdot s} \cdot \frac{K_v}{s} \cdot \frac{1}{N}$$

The angle of open-loop gain is

$$AngleGH(\omega_u) = atan(\omega u\_\omega z) - 180deg$$

and the phase margin is thus given by

$$PM(\omega u\_\omega z) := atan(\omega u\_\omega z)$$

Use to solve for the zero/unity gain bandwidth spacing: $\omega u\_\omega z$

$$\omega u\_\omega z := tan(PM_{des})$$

and the zero location

$$\omega_z := \frac{\omega_u}{\omega u\_\omega z} \qquad \frac{\omega_z}{2 \cdot \pi} = 0.364\,kHz$$

The magnitude transfer function is given by:

$$MagGH(\omega_u) = 1 = \frac{\sqrt{\omega u\_\omega z^2 + 1}}{C_1 \cdot \omega_u} \cdot \frac{K_v}{\omega_u} \cdot \frac{K_\phi}{N}$$

and set this equal to one to find C_1, given ω_u.

$$C_1 := \sqrt{\omega u\_\omega z^2 + 1} \cdot \frac{K_v}{\omega_u^2} \cdot \frac{K_\phi}{N} \qquad C_1 = 370.304\,nF$$

Use C_1 and ω_z to find R_1:

$$R_1 = \frac{1}{\omega_z \cdot C_1} \qquad R_1 := \frac{\omega u\_\omega z}{\sqrt{\omega u\_\omega z^2 + 1} \cdot \frac{K_v}{\omega_u} \cdot \frac{K_\phi}{N}} \qquad R_1 = 1.181\,k\Omega$$

Plugging these back into the transfer function we get: the following feedforward and feedback transfer functions.

$$G(s) = \frac{\omega u\_\omega z \cdot \frac{s}{\omega_u} + 1}{\sqrt{\omega u\_\omega z^2 + 1}} \cdot \frac{N}{\left(\frac{s}{\omega_u}\right)^2} \qquad H := \frac{1}{N}$$

For frequencies much greater than the unity gain bandwidth the magnitude response is

$$G(f) = K_\phi \cdot \frac{R_1 \cdot K_v}{2 \cdot \pi \cdot f} = \frac{\omega u\_\omega z}{\sqrt{\omega u\_\omega z^2 + 1}} \cdot \frac{N}{\left(\frac{f}{f_u}\right)} \qquad H = \frac{1}{N}$$

For $\omega u\_\omega z^2 >> 1$, which is usually true:

$$G(f) = N \cdot \frac{f_u}{f} \qquad H = \frac{1}{N}$$

Note that second order PLL only gives a first order roll off, because the stabilizing zero in the loop.

2^{nd}-Order PLL Design Function

The following function repeats the calculations above

$$pll2nd(f_r, f_o, I, f_u, K_v, PM) := \left| \begin{array}{l} \omega u\_\omega z \leftarrow \tan(PM) \\ N \leftarrow \dfrac{f_o}{f_r} \\ K_\phi \leftarrow \dfrac{I}{2 \cdot \pi \cdot rad} \\ \omega_u \leftarrow 2 \cdot \pi \cdot rad \cdot f_u \\ C_1 \leftarrow \dfrac{\sqrt{\omega u\_\omega z^2 + 1} \cdot K_\phi \cdot K_v}{\omega_u{}^2 \cdot N} \\ R_1 \leftarrow \dfrac{\omega u\_\omega z}{\omega_u \cdot C_1} \\ \begin{pmatrix} \dfrac{C_1}{farad} \\ \dfrac{R_1}{ohm} \end{pmatrix} \end{array} \right.$$

$x := pll2nd(f_r, f_o, I, f_u, K_v, PM_{des})$

$C_1 := x_1 \cdot farad$	$C_1 = 370.304\,nF$	Main Loop Filter Capacitor
$R_1 := x_2 \cdot ohm$	$R_1 = 1.181\,kohm$	Main Loop Resistor

Optimal Settling-Time for 2nd Order PLLs

The closed-loop transfer function for a second order PLL is:

$$A(s) = \frac{N\cdot\left(1 + \omega u\_\omega z\cdot\frac{s}{\omega_u}\right)}{b\cdot\left(\frac{s}{\omega_u}\right)^2 + \omega u\_\omega z\cdot\frac{s}{\omega_u} + 1} \qquad \text{where} \quad b = \left(\omega u\_\omega z^2 + 1\right)^{\frac{1}{2}}$$

To simplified the transient response, we can assume $\omega_u{}^2$ is much greater than $\omega_z{}^2$. In this case the closed loop transfer function simplifies to:

$$A(s) = \frac{N\cdot\left(1 + \omega u\_\omega z\cdot\frac{s}{\omega_u}\right)}{\omega u\_\omega z\cdot\left(\frac{s}{\omega_u}\right)^2 + \omega u\_\omega z\cdot\left(\frac{s}{\omega_u}\right) + 1}$$

The simplified transfer function is acceptable for most situations, except when optimizing the bandwidth. It tends to underestimate the settling time for low phase margins.The inverse Laplace transform of the true transfer function is

$$f_{err}(\omega ut, \omega u\_\omega z) := \begin{array}{|l} b \leftarrow \sqrt{\omega u\_\omega z^2 + 1} \\ c \leftarrow \sqrt{4\cdot\sqrt{\omega u\_\omega z^2 + 1} - \omega u\_\omega z^2} \\ -\omega_u\cdot\exp\left[\frac{-\omega u\_\omega z}{2}\cdot\left(-2 + 5^{\frac{1}{2}}\right)\cdot\omega ut\right]\cdot\frac{\omega ut - \omega u\_\omega z}{\left(\frac{\omega u\_\omega z}{2}\right)^2} \quad \text{if } \omega u\_\omega z = 2\cdot\sqrt{2 + \sqrt{5}} \\ \left[f_{step}\cdot\left[\left(\frac{\omega u\_\omega z}{c}\cdot\sin\left(c\cdot\frac{\omega ut}{2\cdot b}\right) - \cos\left(c\cdot\frac{\omega ut}{2\cdot b}\right)\right)\cdot\exp\left(-\omega u\_\omega z\cdot\frac{\omega ut}{2\cdot b}\right)\right]\right] \quad \text{if } \omega u\_\omega z \neq 2\cdot\sqrt{2 + \sqrt{5}} \end{array}$$

The inverse Laplace transform of the simplified transfer function is

$$f_{err2}(\omega ut, \omega u\_\omega z) := \left[\begin{array}{|l} y \leftarrow \sqrt{\frac{4}{\omega u\_\omega z} - 1} \\ f_{step}\cdot\left(\frac{\omega ut}{2} - 1\right)\cdot\exp\left(\frac{-1}{2}\cdot\omega ut\right) \quad \text{if } \omega u\_\omega z = 4 \\ f_{step}\cdot\left[\left(-\cos\left(\frac{1}{2}\cdot y\cdot\omega ut\right) + \frac{\sin\left(\frac{1}{2}\cdot y\cdot\omega ut\right)}{y}\right)\cdot\exp\left(\frac{-1}{2}\cdot\omega ut\right)\right] \quad \text{if } \omega u\_\omega z \neq 4 \end{array}\right]$$

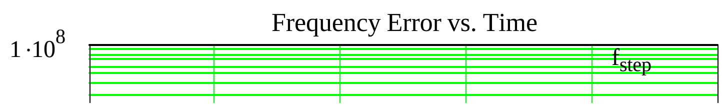
For comparison, we also plot an ideal settling with a bandwidth of ω_u.

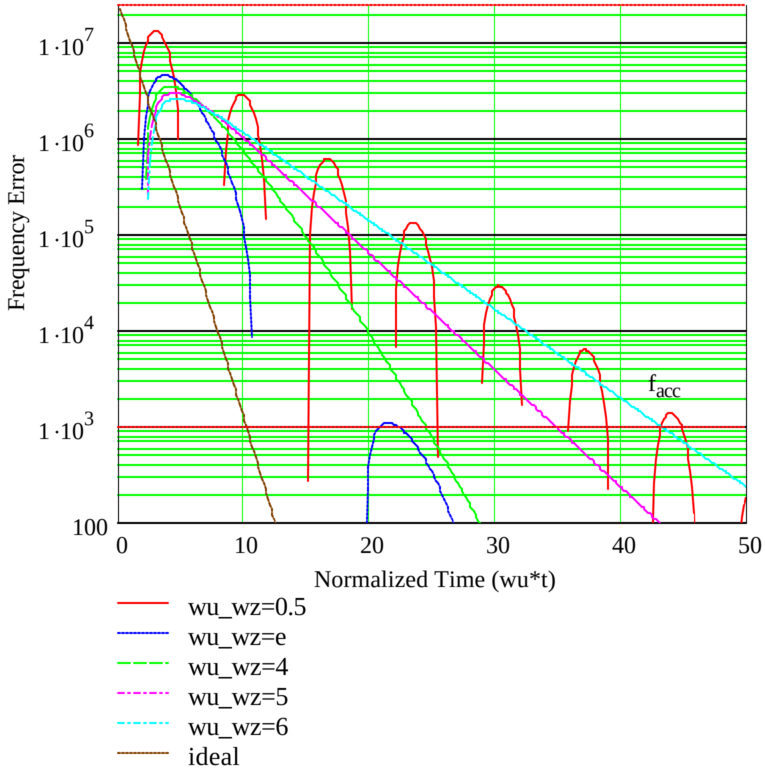
$$f_{errideal}(\omega ut) := f_{step}\cdot e^{-\omega ut}$$

A plot of this response is given below for different values of ωu_ωz.

$num := 400 \qquad i := 1..num$

$$\omega ut_i := \frac{i}{num}\cdot 50$$





The following function calculates the settling time of the transient response

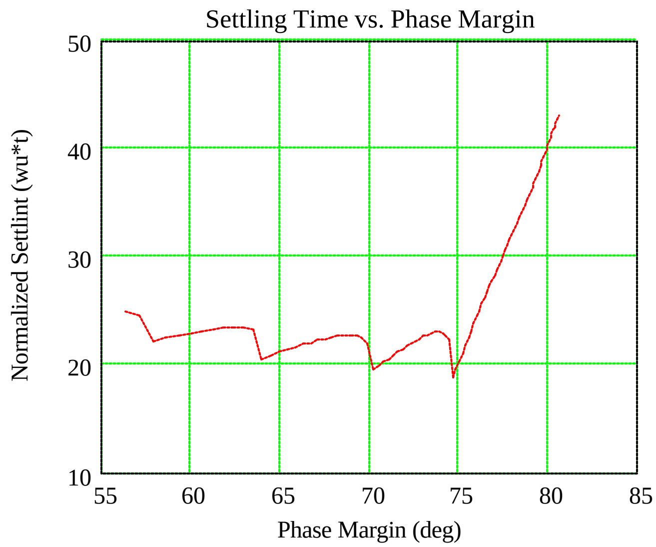
$$t_{settle}(wu\_wz) := \left| \begin{array}{l} \omega ut_{val} \leftarrow 30 \\ \text{for } i \in 2..num \\ \quad \omega ut_{val} \leftarrow \text{if}\left[\left(\left|f_{err}(\omega ut_i, wu\_wz)\right| \le f_{acc}\right)\cdot\left(\left|f_{err}(\omega ut_{i-1}, wu\_wz)\right| > f_{acc}\right), \omega ut_i, \omega ut_{val}\right] \\ \omega ut_{val} \end{array} \right.$$

A plot of the settling time vs phase margin is given below.

$numB := 100$ $\omega u\_\omega z_{min} := 1.5$ $\omega u\_\omega z_{max} := 6$

$i := 1..\, numB$

$$\omega u\_\omega zval_i := \frac{i-1}{numB-1}\cdot\left(\omega u\_\omega z_{max} - \omega u\_\omega z_{min}\right) + \omega u\_\omega z_{min}$$



The following function sweeps through values of wu_wz to find the optimal value.

$$\omega u\_\omega z_{opt} := \left| \begin{array}{l} \omega ut_{setmin} \leftarrow 10^9 \\ \omega u\_\omega z_{opt} \leftarrow 0 \end{array} \right.$$

$\omega u\_\omega z_{opt} \leftarrow 0$

for $i \in 1..numB$

$$\omega u\_\omega z \leftarrow \frac{i-1}{numB-1} \cdot (\omega u\_\omega z_{max} - \omega u\_\omega z_{min}) + \omega u\_\omega z_{min}$$

$\omega ut_{settle} \leftarrow$
- $\omega ut_{val} \leftarrow 10^{9} sec$
- for $i \in 2..num$
  - $\omega ut_{val} \leftarrow if[(|f_{err}(\omega ut_i, \omega u\_\omega z)| \le f_{acc}) \cdot (|f_{err}(\omega ut_{i-1}, \omega u\_\omega z)| > f_{acc}), \omega ut_i, \omega ut_{val}]$
- ωut_{val}

$\omega u\_\omega z_{opt} \leftarrow if(\omega ut_{settle} < \omega ut_{setmin}, \omega u\_\omega z, \omega u\_\omega z_{opt})$

$\omega ut_{setmin} \leftarrow if(\omega ut_{settle} < \omega ut_{setmin}, \omega ut_{settle}, \omega ut_{setmin})$

$\omega u\_\omega z_{opt}$

$\omega u\_\omega z_{opt} = 3.636$

This corresponds to an optimum phase margin of

$PM_{opt} := PM(\omega u\_\omega z_{opt})$ $\qquad$ $PM_{opt} = 74.624 deg$

We decrease this phase margin slightly to account for loop gain variations. The optimal settling time is

$t_{settleopt} := t_{settle}(\omega u\_\omega z_{opt})$ $\qquad$ $t_{settleopt} = 18.75$

Outputs

$C_1 = 370.304 nF$ $\qquad$ Main Loop Filter Capacitor

$R_1 = 1.181 kohm$ $\qquad$ Main Loop Resistor

Example Noise Parameters

Using typical phase noise values, we can plot the phase noise of the overall loop.

$L_{vco} := -110\,dBC\_Hz$ — VCO Phase Noise at f_{vco}

$f_{vco} := 100\,kHz$ — Frequency for VCO Phase Noise

$L_{pdet} := -120\,dBC\_Hz$ — Reference Phase Noise at f_{pdet}

$f_{pdet} := 100\,kHz$ — Frequency for Phase Detector Phase Noise

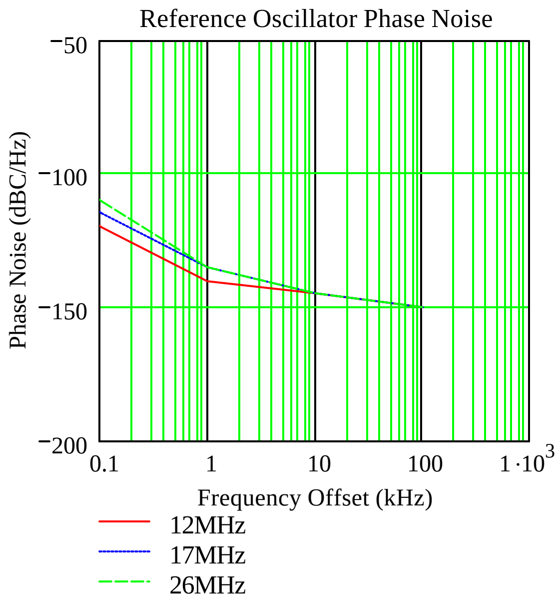
$f_{start} := \frac{f_u}{10}$ $\quad f_{stop} := f_u \cdot 1000$ — Start and Stop Frequencies for Plotting

Measured Lref data from http://www.rakon.com/VTXO100spec.html

$$f_{refMHz} := \begin{pmatrix} 1 \\ 10 \\ 100 \\ 1000 \\ 10^4 \\ 10^5 \end{pmatrix} Hz \quad L_{ref12MHz} := \begin{pmatrix} -60 \\ -90 \\ -120 \\ -140 \\ -145 \\ -150 \end{pmatrix} dBC\_Hz \quad L_{ref17MHz} := \begin{pmatrix} -55 \\ -85 \\ -115 \\ -135 \\ -145 \\ -150 \end{pmatrix} dBC\_Hz \quad L_{ref26MHz} := \begin{pmatrix} -50 \\ -80 \\ -110 \\ -135 \\ -145 \\ -150 \end{pmatrix} dBC\_Hz$$

$i := 1 .. num \quad f_i := f_{start} \cdot \left(\frac{f_{stop}}{f_{start}} \right)^{\frac{i}{num}}$ — Logarithmic Frequency Vector

$L_{ref_i} := \text{linterp}\left(\overrightarrow{\log\left(\frac{f_{refMHz}}{Hz}\right)}, L_{ref12MHz}, \log\left(\frac{f_i}{Hz}\right) \right)$ — Log-Linear Interpolated Reference Phase Noise Vector



Small Signal Transfer Functions

$\omega_i := 2\cdot\pi\cdot f_i \qquad s_i := j\cdot\omega_i$

The loop's feedforward and feedback transfer functiosn are

$$G_i := \frac{K_\phi\cdot\left(1 + R_1\cdot C_1\cdot s_i\right)}{C_1\cdot s_i}\cdot\frac{K_v}{s_i}$$ Open Loop Feedforward Gain

$$H := \frac{1}{N}$$ Open Feedback Gain

When the loop is modulated, it is useful to know the group delay of the PLL as a function of frequency.

$$gdhp(f) := \frac{d}{df}\arg\left[\frac{1}{1 + \frac{K_\phi\cdot\left(1 + R_1\cdot C_1\cdot j\cdot 2\cdot\pi\cdot f\right)}{C_1\cdot j\cdot 2\cdot\pi\cdot f}\cdot\frac{K_v}{j\cdot 2\cdot\pi\cdot f}\cdot\frac{1}{N}}\right]$$ High Pass Group Delay

$$gdlp(f) := \frac{d}{df}\arg\left[\frac{\frac{K_\phi\cdot\left(1 + R_1\cdot C_1\cdot j\cdot 2\cdot\pi\cdot f\right)}{C_1\cdot j\cdot 2\cdot\pi\cdot f}\cdot\frac{K_v}{j\cdot 2\cdot\pi\cdot f}}{1 + \frac{K_\phi\cdot\left(1 + R_1\cdot C_1\cdot j\cdot 2\cdot\pi\cdot f\right)}{C_1\cdot j\cdot 2\cdot\pi\cdot f}\cdot\frac{K_v}{j\cdot 2\cdot\pi\cdot f}\cdot\frac{1}{N}}\right]$$ Low Pass Group Delay

Phase Noise Calculations

The feedforward noise transfer function for R_1 to the output is:

$G_{R1_i} := 1$

The closed loop noise transfer function for R_1 to the output is:

$$L_{R1_i} := \left[4\cdot k\cdot Temp\cdot R_1\cdot\left(\left|G_{R1_i}\right|\right)^2\right]\cdot\left[\left|\frac{K_v}{\left(1 + G_i\cdot H\right)\cdot s_i}\right|\right]^2$$

The phase noise spectrum of VCO to the output is:

$$L_{VCO_i} := \left(\frac{10^{\frac{L_{vco} - 20\cdot\log\left(\frac{f_i}{f_{vco}}\right)}{10}}}{Hz}\right)\cdot\left(\left|\frac{1}{1 + G_i\cdot H}\right|\right)^2$$

The phase noise spectrum of reference oscillator to the output is:

$$L_{REF_i} := \frac{10^{\frac{L_{ref_i}}{10}}}{Hz}\cdot\left(\left|\frac{G_i}{1 + G_i\cdot H}\right|\right)^2$$

Phase noise spectrum of phase detector to the output is:

$$L_{PDET_i} := \frac{10^{\frac{L_{pdet} - 10\cdot\log\left(\frac{f_i}{f_{pdet}}\right)}{10}}}{Hz}\cdot\left(\left|\frac{G_i}{1 + G_i\cdot H}\right|\right)^2$$

The total output phase noise spectrum is

$$L_{tot_i} := 10\cdot\log\left[\left(L_{R1_i} + L_{VCO_i} + L_{PDET_i} + L_{REF_i}\right)\cdot Hz\right]$$

Transient Step Response

Here two transient responses for the PLL are plotted, one with the effects of slewing considered and one without. The maximum slew-rate for the output frequency is limited by the rate at which the charge-pump can charge the loop filter capacitor.

$$\Delta f\_\Delta t_{max} := \frac{I}{C_1} \cdot \frac{K_v}{2 \cdot \pi} \qquad \Delta f\_\Delta t_{max} = 1.35 \frac{MHz}{mS}$$

A pure slewing step response ise given by the following expression:

$$h_{slew}(t) := \Delta f\_\Delta t_{max} \cdot t$$

Time to slew to desired frequency

$$t_{slew} := \frac{f_{step}}{\Delta f\_\Delta t_{max}} \qquad t_{slew} = 18.515\,mS$$

The slewing expression is interesting to see when all the variables have been substituted. Here we see that increasing the bandwidth (fractional-N), reducing the frequency step size (with switch-cap tuning), and higher phase margins can improve the slew rate, and increasing the output frequency (which also impacts fstep). In general slewing improvement requires topology changes, not just a resizing.

$$t_{slew} = \frac{f_{step} \cdot \omega u\_\omega z}{N \cdot {\omega_u}^2}$$

A pure linear step response and frequency error is given by the following expressions:

$$h(t) := f_{step} + f_{err}(\omega_u \cdot t, \omega u\_\omega z)$$

$$f_{linerr}(t) := f_{step} - h(t)$$

To combine the linear and slewing responses, we find the time where the slopes are equal:

$$t_{lin} := \left| \begin{array}{l} t \leftarrow \frac{\pi}{\omega_u} \\ \mathrm{root}\left(\frac{d}{dt} h(t) - \Delta f\_\Delta t_{max}, t\right) \end{array} \right. \qquad t_{lin} = 0.562\,mS$$

Time where linear slewing starts

Time where slewing stops

$$t_{slew} := \frac{h(t_{lin})}{\Delta f\_\Delta t_{max}} \qquad t_{slew} = 21.817\,mS$$

The linear + slewing step response is given as:

$$h_{linslew}(t) := \mathrm{if}(t < t_{slew}, h_{slew}(t), h(t - t_{slew} + t_{lin}))$$

with a frequency error of

$$f_{linslewerr}(t) := f_{step} - h_{linslew}(t)$$

The overall settling time is then calculated

$num := 1000 \qquad i := 1..\,num$

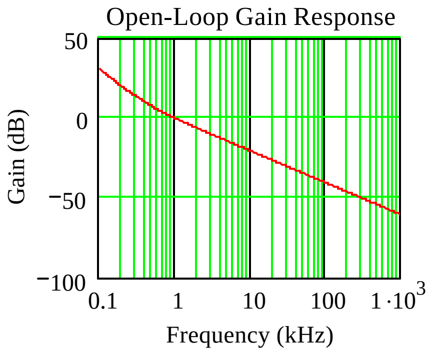
$$t_i := \frac{i}{num} \cdot \left(\frac{50}{\omega_u} + t_{slew}\right)$$

Time Vector

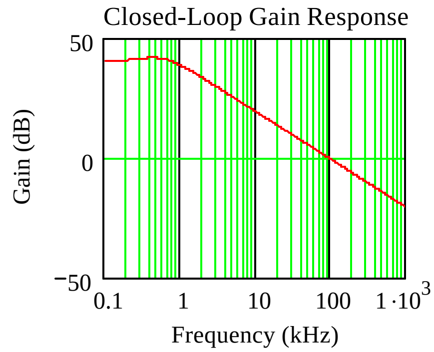
$$t_{settle} := \left| \begin{array}{l} t_{val} \leftarrow 0sec \\ \text{for } i \in 2..\,num \\ \quad t_{val} \leftarrow \mathrm{if}\left[\left(\left|f_{linslewerr}(t_i)\right| \le f_{acc}\right) \cdot \left(\left|f_{linslewerr}(t_{i-1})\right| > f_{acc}\right), t_i, t_{val}\right] \\ t_{val} \end{array} \right.$$

$t_{settle} = 24.326\,mS$ Total Settling Time

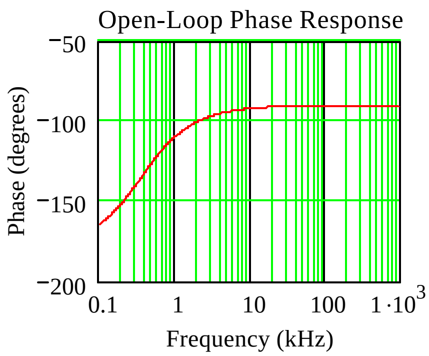
Plots



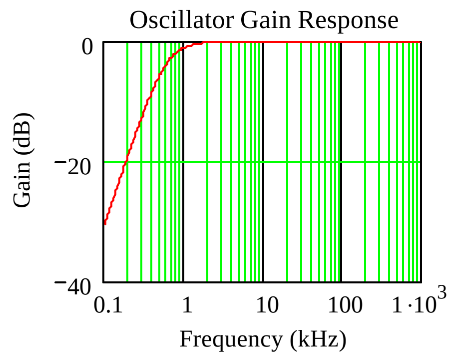
Open-Loop Gain Response
50
0
−50
−100
Gain (dB)
0.1
1
10
100
1 ·10^3
Frequency (kHz)



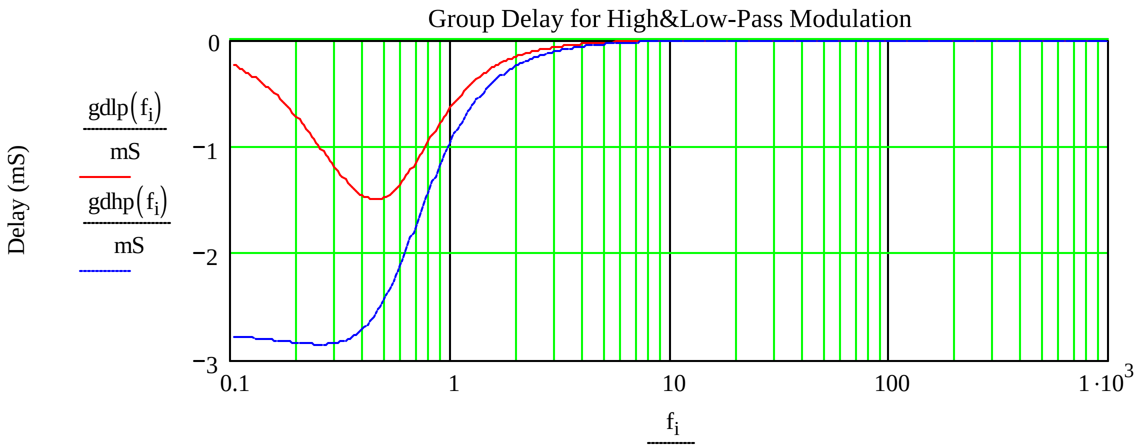
Closed-Loop Gain Response
50
0
−50
Gain (dB)
0.1
1
10
100
1 ·10^3
Frequency (kHz)

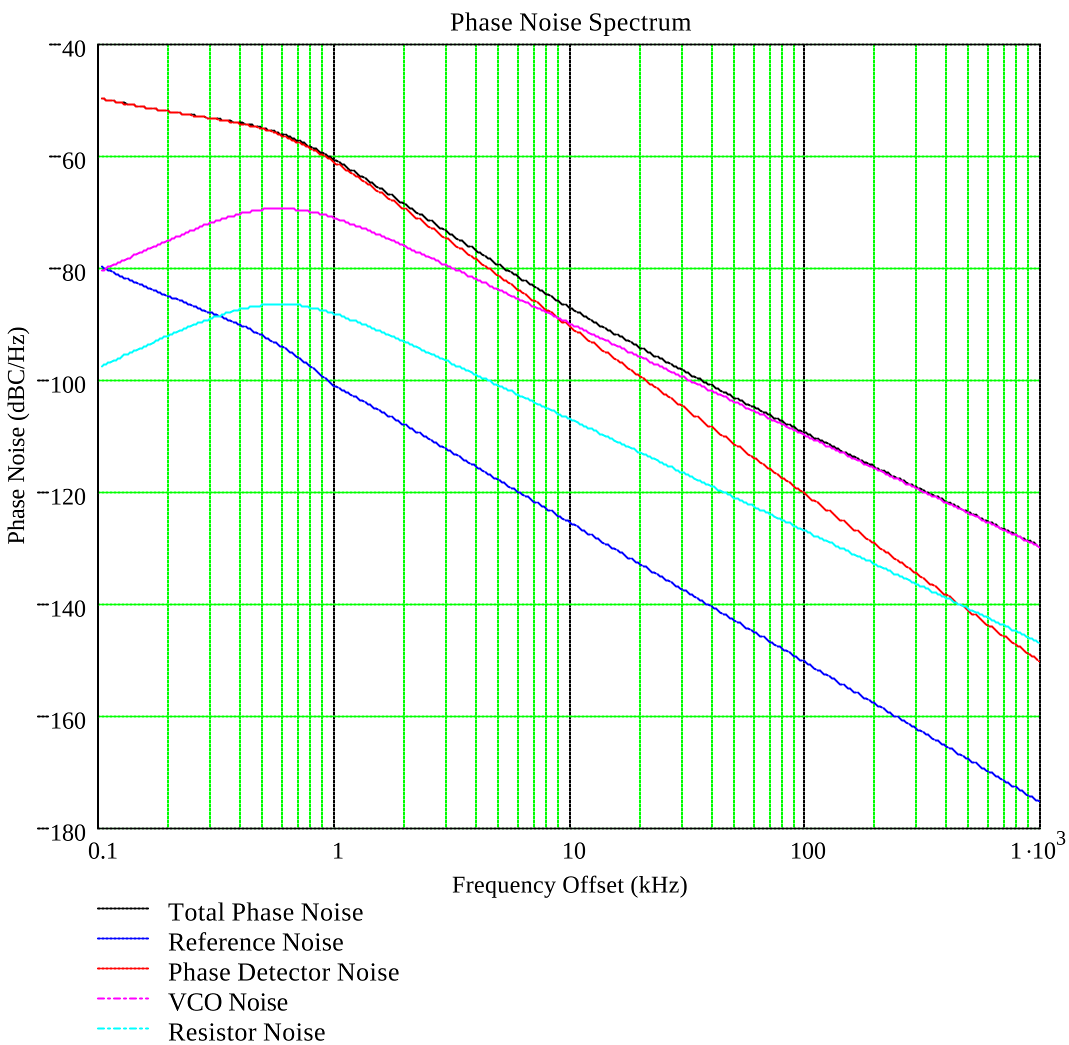


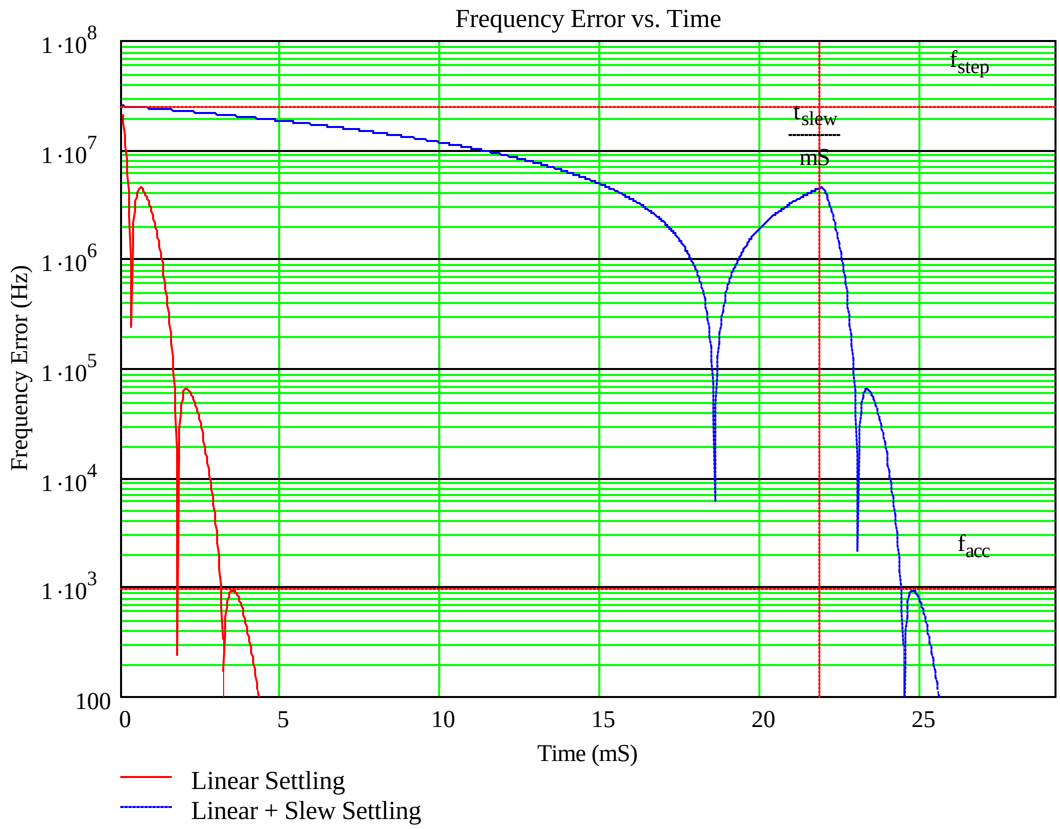
Open-Loop Phase Response
−50
−100
−150
−200
Phase (degrees)
0.1
1
10
100
1 ·10^3
Frequency (kHz)

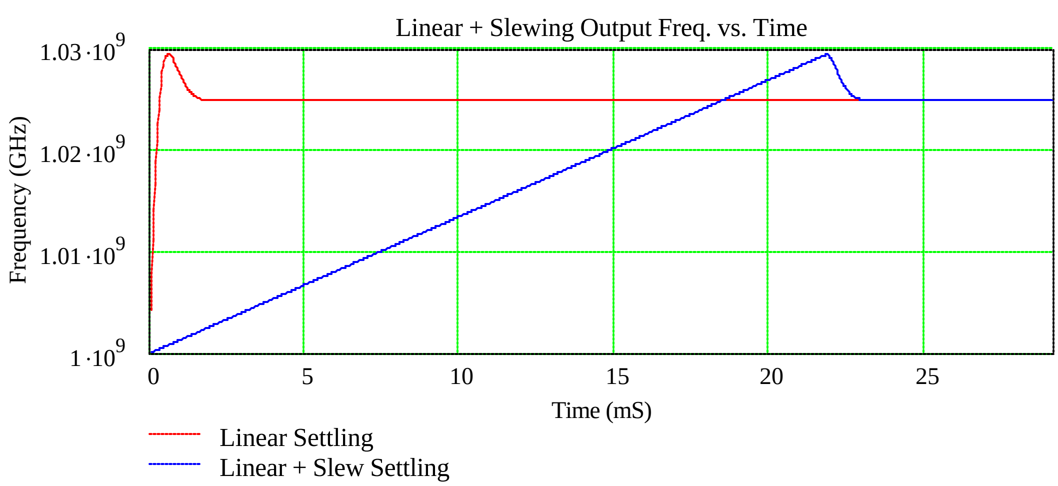


Oscillator Gain Response
0
−20
−40
Gain (dB)
0.1
1
10
100
1 ·10^3
Frequency (kHz)


Group Delay for High&Low-Pass Modulation
Delay (mS)
gdlp(fi) / mS
gdhp(fi) / mS
0
−1
−2
−3
0.1
1
10
100
1·10^3
fi


Phase Noise Spectrum
Phase Noise (dBC/Hz)
−40
−60
−80
−100
−120
−140
−160
−180
0.1
1
10
100
1·10^3
Frequency Offset (kHz)
Total Phase Noise
Reference Noise
Phase Detector Noise
VCO Noise
Resistor Noise





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