2nd Order PLL Design and Analysis

Fig. 1: 2nd Order PLL with Current-Mode Phase Detector

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**Introduction**

Although the emphasis of PLL design is on 3rd-order PLLs, the settling time of second order PLLs is important, especially for boosted PLLs. In boost mode the transfer function of a third-order PLL sometimes reverts to a second order expression. As with 3rd-order PLL's there are different implementations, including voltage mode and those with operational-amplifier based phase detectors. This report will talk about the most popular, which uses a current-mode phase detector and a passive loop filter.

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**Inputs**

- $f_r := 10\text{ MHz}$
- $f_o := 1\text{ GHz}$
- $f_{acc} := 1\text{ kHz}$
- $f_{step} := 25\text{ MHz}$
- $I := 50\mu\text{A}$
- $f_u := 1\text{ kHz}$
- $K_v := 2\pi \times 10^{-6}\frac{\text{rad}}{\text{sec-volt}}$
- $\text{PM}_{des} := 70\text{deg}$

**Initial Calculations**

- $T := \frac{1}{f_r}$
- $\omega_u := 2\pi f_u$
- $N := \frac{f_o}{f_r}$
- $K_\phi := \frac{1}{2\pi}$
- $T = 0.1\mu\text{S}$
- $\omega_u = 6.283 \times 10^3 \frac{\text{rad}}{\text{sec}}$
- $N = 100$
- $K_\phi = 7.958 \frac{\mu\text{A}}{\text{rad}}$

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**Reference oscillator frequency and period**

**Output frequency**

**Acceptable Frequency Error**

**Maximum Frequency Step (output referred)**

**Charge Pump Current**

**Unity Gain Frequency**

**VCO Gain**

**Phase Margin** (70 degrees recommended, 50 degrees if slewing is considered)
The open-loop transfer functions for the PLL are:
\[ \text{GH}(s) = K_\phi \frac{R_1 C_1 s + 1}{C_1 s} \frac{K_v}{s} = K_\phi \frac{\omega_u s + 1}{C_1 s} \frac{K_v}{s} \]

The angle of open-loop gain is
\[ \text{AngleGH}(\omega_u) = \arctan(\omega_u / \omega_z) - 180 \text{deg} \]
and the phase margin is thus given by
\[ \text{PM}(\omega_u / \omega_z) = \arctan(\omega_u / \omega_z) \]

Use to solve for the zero/unity gain bandwidth spacing: \( \omega_u / \omega_z \)
\[ \omega_u / \omega_z := \tan(\text{PM}_{\text{des}}) \]

and the zero location
\[ \omega_z := \frac{\omega_u}{\omega_u / \omega_z} \quad \frac{\omega_z}{2 \pi} = 0.364 \text{kHz} \]

The magnitude transfer function is given by:
\[ \text{MagGH}(\omega_u) = 1 = \sqrt{\frac{\omega_u / \omega_z^2 + 1}{\omega_u / \omega_z^2} \frac{K_v}{K_\phi} N} \]

and set this equal to one to find \( C_1 \), given \( \omega_u \).
\[ C_1 := \sqrt{\frac{\omega_u / \omega_z^2 + 1}{\omega_u / \omega_z^2} \frac{K_v}{K_\phi} N} \quad \text{C}_1 = 370.304 \text{nF} \]

Use \( C_1 \) and \( \omega_z \) to find \( R_1 \):
\[ R_1 = \frac{1}{\omega_z C_1} \quad R_1 := \frac{\omega_u / \omega_z}{\sqrt{\frac{\omega_u / \omega_z^2 + 1}{\omega_u / \omega_z^2} \frac{K_v}{K_\phi} N}} \quad \text{R}_1 = 1.181 \text{k} \Omega \]

Plugging these back into the transfer function we get: the following feedforward and feedback transfer functions.
\[ G(s) = \frac{\omega_u / \omega_z s + 1}{\sqrt{\omega_u / \omega_z^2 + 1} \left( \frac{s}{\omega_u} \right)^2} \quad H := \frac{1}{N} \]

For frequencies much greater than the unity gain bandwidth the magnitude response is
\[ G(f) = K_\phi \frac{R_1 \cdot K_v}{2 \pi f} = \frac{\omega_u / \omega_z}{\sqrt{\omega_u / \omega_z^2 + 1} \left( \frac{f}{f_u} \right)} \quad H = \frac{1}{N} \]

For \( \omega_u / \omega_z^2 >> 1 \), which is usually true:
\[ G(f) = \frac{f_u}{f} \quad H = \frac{1}{N} \]

Note that second order PLL only gives a first order roll off, because the stabilizing zero in the loop.
2nd-Order PLL Design Function

The following function repeats the calculations above

\[
\text{pll2nd}(f_r, f_0, I, f_u, K_v, PM) := \begin{align*}
\omega u_\omega & \leftarrow \tan(\text{PM}) \\
N & \leftarrow \frac{f_0}{f_r} \\
K_\phi & \leftarrow \frac{1}{2\pi \text{ rad}} \\
\omega_u & \leftarrow 2\pi \text{ rad} \cdot f_u \\
C_1 & \leftarrow \frac{\sqrt{\omega u_\omega^2 + 1 \cdot K_\phi \cdot K_v}}{\omega_u^2 \cdot N} \\
R_1 & \leftarrow \frac{\omega u_\omega}{\omega_u \cdot C_1} \\
\begin{pmatrix} C_1 & R_1 \end{pmatrix} & \begin{pmatrix} \text{farad} & \text{ohm} \end{pmatrix}
\end{align*}
\]

\[x := \text{pll2nd}(f_r, f_0, I, f_u, K_v, \text{PM}_{\text{des}})\]

\[C_1 := x_1 \text{ farad} \quad C_1 = 370.304 \text{ nF} \quad \text{Main Loop Filter Capacitor}\]

\[R_1 := x_2 \text{ ohm} \quad R_1 = 1.181 \text{ kohm} \quad \text{Main Loop Resistor}\]
**Optimal Settling-Time for 2nd Order PLLs**

The closed-loop transfer function for a second order PLL is:

\[
A(s) = \frac{N \left(1 + \frac{s}{\omega_u} - \frac{s}{\omega_z} \right)}{\omega_u \left(\frac{s}{\omega_u} \right)^2 + \omega_u \left(\frac{s}{\omega_u} \right) + 1}
\]

where \( b = \left(\frac{\omega_u}{\omega_z} + 1\right) \)

To simplify the transient response, we can assume \( \omega_u^2 \) is much greater than \( \omega_z^2 \). In this case the closed loop transfer function simplifies to:

\[
A(s) = \frac{N \left(1 + \frac{s}{\omega_u} - \frac{s}{\omega_z} \right)}{\omega_u \left(\frac{s}{\omega_u} \right)^2 + \omega_u \left(\frac{s}{\omega_u} \right) + 1}
\]

The simplified transfer function is acceptable for most situations, except when optimizing the bandwidth. It tends to underestimate the settling time for low phase margins. The inverse Laplace transform of the true transfer function is

\[
f_{err}(\text{out}, \omega_u, \omega_z) := \begin{cases} 
  b \left(\sqrt{\frac{\omega_u^2}{\omega_z^2} + 1}\right) 
  & \text{if } \omega_u \omega_z = 2\sqrt{2 + \sqrt{5}} \\
  c \left(\sqrt{4\omega_u^2 + 1 - \omega_u^2}\right) 
  & \text{if } \omega_u \omega_z \neq 2\sqrt{2 + \sqrt{5}} \\
  -\omega_u \exp\left[\frac{-\omega_u}{2\sqrt{2 + \sqrt{5}}}
  \right] \left(\frac{\omega_u - \omega_z}{2}\right) 
  & \text{if } \omega_u \omega_z = 2\sqrt{2 + \sqrt{5}} \\
  f_{step}\left[\left(\frac{\omega_u}{c} \sin\left(\frac{c \cdot \text{out}}{2\sqrt{2 + \sqrt{5}}}\right) - \cos\left(\frac{c \cdot \text{out}}{2\sqrt{2 + \sqrt{5}}}\right)\right)\exp\left(-(\omega_u - \omega_z) \cdot \frac{\text{out}}{2\sqrt{2 + \sqrt{5}}}\right)\right] 
  & \text{if } \omega_u \omega_z \neq 2\sqrt{2 + \sqrt{5}}
\end{cases}
\]

The inverse Laplace transform of the simplified transfer function is

\[
f_{err2}(\text{out}, \omega_u, \omega_z) := \begin{cases} 
  y \left(\sqrt{\frac{4}{\omega_u^2}} + 1\right) 
  & \text{if } \omega_u \omega_z = 4 \\
  f_{step}\left(\frac{\text{out}}{2} - 1\right) \exp\left(-\frac{1}{2} \cdot \text{out}\right) 
  & \text{if } \omega_u \omega_z \neq 4 \\
  f_{step}\left[-\cos\left(\frac{1}{2} \cdot \text{out}\right) + \sin\left(\frac{1}{2} \cdot \text{out}\right)\right] \exp\left(\frac{1}{2} \cdot \text{out}\right) 
  & \text{if } \omega_u \omega_z \neq 4
\end{cases}
\]

For comparison, we also plot an ideal settling with a bandwidth of \( \omega_u \). 

\[
f_{err\text{ideal}}(\text{out}) := f_{step} e^{-\text{out}}
\]

A plot of this response is given below for different values of \( \omega_u, \omega_z \).

\[
\text{num} := 400 \\
i := 1..\text{num}
\]

\[
\text{out}_i := \frac{i \cdot \text{num}}{\text{num}}
\]

Frequency Error vs. Time

1.0 - 8

\[ f_{step} \]
The following function calculates the settling time of the transient response:

\[
t_{\text{settle}}(wu_{\omega z}) := \begin{cases} 
\omega_{\text{ut val}} & \leq 30 \\
\text{for } i \in 2.. \text{num} \\
\omega_{\text{ut val}} & \leftarrow \text{if} \left( \left| f_{\text{err}}(\omega_{\text{ut}}, wu_{\omega z}) \right| \leq f_{\text{acc}} \right) \left( \left| f_{\text{err}}(\omega_{\text{ut} - 1}, wu_{\omega z}) \right| > f_{\text{acc}} \right), \omega_{\text{ut} i}, \omega_{\text{ut val}} \right)
\end{cases}
\]

A plot of the settling time vs phase margin is given below.

The following function sweeps through values of \( wu_{\omega z} \) to find the optimal value.

\[
\omega_{\text{u opt}} := \begin{cases} 
\omega_{\text{ut setmin}} & \leftarrow 10^9 \\
\text{for } i \in 1.. \text{numB} \\
\omega_{\text{ut val i}} & := \frac{i - 1}{\text{numB}} \left( \omega_{\text{u max}} - \omega_{\text{u min}} \right) + \omega_{\text{u min}}
\end{cases}
\]
for $i \in 1..\text{numB}$

\[
\omega_u_{\omega z} \leftarrow \frac{i - 1}{\text{numB} - 1} \left( \omega_u_{\omega z_{\text{max}}} - \omega_u_{\omega z_{\text{min}}} \right) + \omega_u_{\omega z_{\text{min}}}
\]

\[
\omega_{\text{out settle}} \leftarrow \omega_{\text{out val}} \leftarrow 10^9 \text{ sec}
\]

for $i \in 2..\text{num}$

\[
\omega_{\text{out val}} \leftarrow \begin{cases} 
\omega_{\text{out val}} & \text{if } \left( |f_{\text{err}}(\omega_{\text{out i}}, \omega_u_{\omega z})| \leq f_{\text{acc}} \right) \cdot \left( |f_{\text{err}}(\omega_{\text{out i-1}}, \omega_u_{\omega z})| > f_{\text{acc}} \right) \cdot \omega_{\text{out i}} \cdot \omega_{\text{out val}} 
\end{cases}
\]

\[
\omega_{\omega_2 \omega_2 \omega_2} \leftarrow \text{if } (\omega_{\text{out settle} < \omega_{\text{out settle min}}, \omega_u_{\omega z}, \omega_u_{\omega z_{\text{opt}}})
\]

\[
\omega_{\text{out settle min}} \leftarrow \text{if } (\omega_{\text{out settle} < \omega_{\text{out settle min}}, \omega_{\text{out settle}}, \omega_{\text{out settle min}})
\]

This corresponds to an optimum phase margin of

\[
\text{PM}_{\text{opt}} \ := \text{PM}(\omega_u_{\omega z_{\text{opt}}})
\]

\[
\text{PM}_{\text{opt}} = 74.624 \text{ deg}
\]

We decrease this phase margin slightly to account for loop gain variations. The optimal settling time is

\[
\text{t}_{\text{settle opt}} := t_{\text{settle}}(\omega_u_{\omega z_{\text{opt}}})
\]

\[
\text{t}_{\text{settle opt}} = 18.75
\]

**Outputs**

- $C_1 = 370.304 \text{nF}$
  - Main Loop Filter Capacitor
- $R_1 = 1.181 \text{kohm}$
  - Main Loop Resistor
Example Noise Parameters

Using typical phase noise values, we can plot the phase noise of the overall loop.

\[
L_{\text{vco}} := -110 \text{dBC}_\text{Hz}
\]

\[
f_{\text{vco}} := 100\text{kHz}
\]

\[
L_{\text{pdet}} := -120 \text{dBC}_\text{Hz}
\]

\[
f_{\text{pdet}} := 100\text{kHz}
\]

\[
f_{\text{start}} := \frac{f_u}{10} \quad f_{\text{stop}} := f_u \cdot 1000
\]

Measured Lref data from http://www.rakon.com/VTXO100spec.html

\[
\begin{array}{c|c|c|c|c}
\hline
f_{\text{refMHz}} & L_{\text{ref}12\text{MHz}} & L_{\text{ref}17\text{MHz}} & L_{\text{ref}26\text{MHz}} \\
\hline
1 & -60 & -55 & -50 \\
10 & -90 & -85 & -80 \\
100 & -120 & -115 & -110 \\
10^4 & -140 & -135 & -135 \\
10^5 & -145 & -145 & -145 \\
\hline
\end{array}
\]

\[
\text{Log-Linear Interpolated Reference Phase Noise Vector}
\]

\[
L_{\text{ref}_{i \text{interp log}}} := \log \left( \frac{f_{\text{refMHz}}}{\text{Hz}} \right) \cdot L_{\text{ref}12\text{MHz},i} \cdot \log \left( \frac{f}{\text{Hz}} \right)
\]

\[
\text{Log-Linear Interpolated Reference Phase Noise Vector}
\]

Reference Oscillator Phase Noise

Using typical phase noise values, we can plot the phase noise of the overall loop.
Small Signal Transfer Functions

\[ \omega_i := 2 \pi f_i \quad s_i := j \omega_i \]

The loop's feedforward and feedback transfer functions are

\[
G_i := \frac{K_\phi}{C_i s_i} \quad \text{Open Loop Feedforward Gain}
\]

\[
H := \frac{1}{N} \quad \text{Open Feedback Gain}
\]

When the loop is modulated, it is useful to know the group delay of the PLL as a function of frequency.

\[
gdhp(f) := \frac{d}{df} \arg \left[ \frac{1}{1 + \frac{K_\phi (1 + R_1 C_i s_i j 2 \pi f) K_v}{C_i s_i j 2 \pi f}} \right] \quad \text{High Pass Group Delay}
\]

\[
gdlp(f) := \frac{d}{df} \arg \left[ \frac{1}{1 + \frac{K_\phi (1 + R_1 C_i s_i j 2 \pi f) K_v}{C_i s_i j 2 \pi f}} \right] \quad \text{Low Pass Group Delay}
\]

Phase Noise Calculations

The feedforward noise transfer function for \( R_1 \) to the output is:

\[ G_{R1} := 1 \]

The closed loop noise transfer function for \( R_1 \) to the output is:

\[ L_{R1} := \left[ 4k \text{Temp} \cdot R_1 \left( \left| G_{R1} \right| \right)^2 \right] \left[ \left| \frac{K_v}{1 + G_i H} \right| s_i \right]^2 \]

The phase noise spectrum of VCO to the output is:

\[ \frac{L_{VCO} := 10 \log \left( \frac{f_i}{f_{vco}} \right)}{10} \left( \frac{1}{1 + G_i H} \right)^2 \]

The phase noise spectrum of reference oscillator to the output is:

\[ L_{REF} := \frac{10 \log \left( \frac{G_i}{1 + G_i H} \right)}{10} \]

Phase noise spectrum of phase detector to the output is:

\[ L_{PDET} := \frac{10 \log \left( \frac{f_i}{f_{pdet}} \right)}{10} \left( \frac{G_i}{1 + G_i H} \right)^2 \]

The total output phase noise spectrum is

\[ L_{tot} := 10 \log \left[ \left( L_{R1} + L_{VCO} + L_{PDET} + L_{REF} \right) \text{Hz} \right] \]
**Transient Step Response**

Here two transient responses for the PLL are plotted, one with the effects of slewing considered and one without. The maximum slew-rate for the output frequency is limited by the rate at which the charge-pump can charge the loop filter capacitor.

\[
\Delta f_{\Delta t_{\text{max}}} := \frac{I}{C_1} \cdot \frac{K_v}{2 \pi} \quad \Delta f_{\Delta t_{\text{max}}} = 1.35 \text{ MHz/mS}
\]

A pure slewing step response is given by the following expression:

\[h_{\text{slew}}(t) := \Delta f_{\Delta t_{\text{max}}} \cdot t\]

Time to slew to desired frequency

\[t_{\text{slew}} := \frac{f_{\text{step}}}{\Delta f_{\Delta t_{\text{max}}}} \quad t_{\text{slew}} = 18.515 \text{ mS}\]

The slewing expression is interesting to see when all the variables have been substituted. Here we see that increasing the bandwidth (fractional-N), reducing the frequency step size (with switch-cap tuning), and higher phase margins can improve the slew rate, and increasing the output frequency (which also impacts fstep). In general slewing improvement requires topology changes, not just a resizing.

\[t_{\text{slew}} = \frac{f_{\text{step}} \frac{\omega_u}{\omega_z}}{N \omega_u^2} \]

A pure linear step response and frequency error is given by the following expressions:

\[h(t) := f_{\text{step}} + f_{\text{err}}(\omega_u, t, \omega_u, \omega_z)\]

\[f_{\text{linerr}}(t) := f_{\text{step}} - h(t)\]

To combine the linear and slewing responses, we find the time where the slopes are equal:

\[t_{\text{lin}} := \text{root} \left( \frac{\pi}{\omega_u} - \frac{\frac{\pi}{\omega_u} \cdot \Delta f_{\Delta t_{\text{max}}} \cdot t}{h(t)} \right) \quad t_{\text{lin}} = 0.562 \text{ mS} \quad \text{Time where linear slewing starts}\]

Time where slewing stops

\[t_{\text{slew}} := \frac{h(t_{\text{lin}})}{\Delta f_{\Delta t_{\text{max}}}} \quad t_{\text{slew}} = 21.817 \text{ mS}\]

The linear + slewing step response is given as:

\[h_{\text{lin}slew}(t) := \begin{cases} h(t) & \text{if } t < t_{\text{slew}}, \text{ otherwise } \end{cases}, h(t - t_{\text{slew}} + t_{\text{lin}})\]

with a frequency error of

\[f_{\text{lin}slewerr}(t) := f_{\text{step}} - h_{\text{lin}slew}(t)\]

The overall settling time is then calculated

\[\text{num} := 1000 \quad i := 1..\text{num}\]

\[t_i := \frac{i}{\text{num}} \left( \frac{50}{\omega_u} + t_{\text{slew}} \right) \quad \text{Time Vector}\]

\[t_{\text{settle}} := \begin{cases} t_{\text{val}} := 0 \text{sec} & \text{for } i \in 2..\text{num} \\ t_{\text{val}} := \text{if } \left[ \left| f_{\text{lin}slewerr}(t_i) \right| \leq f_{\text{acc}} \left( \left| f_{\text{lin}slewerr}(t_{i-1}) \right| > f_{\text{acc}} \right), t_i, t_{\text{val}} \right] & \text{ otherwise } \end{cases}\]

\[t_{\text{settle}} = 24.326 \text{ mS} \quad \text{Total Settling Time}\]
Plots

Open-Loop Gain Response

Closed-Loop Gain Response

Open-Loop Phase Response

Oscillator Gain Response
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