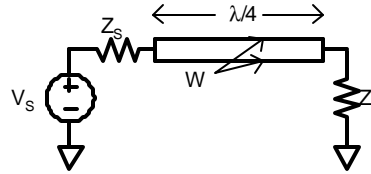


Impedance Matching with Transmission Lines



- ▶ useful functions and identities
- ▶ Units
- ▶ Constants

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Introduction

Transmission lines have some special properties that can make them advantageous for impedance matching. First they can provide an open circuit at certain frequencies, and a short at others, making them useful for biasing, where an impedance needs to be a short at DC, but an open at a desired frequency. This property also allows them to short out harmonics of a signal, without attenuation of the desired signal. At very high frequencies (>2GHz), the physical dimensions of inductors and capacitors make them difficult to manufacture. At these frequencies, inductors and capacitors are replaced with transmission lines, made cheap and small out of PC board.

Inputs

$Z_L := 100\Omega$

$Z_S := 50\Omega$

$S_{11max} := -10\text{dB}$

$f_c := 1\text{GHz}$

$\Delta f := 0.3\text{GHz}$

$\epsilon_r := 4.2$

$h := 60\text{mil}$

Load Impedance

Source Impedance

Maximum Tolerable S_{11}

Center Frequency

Bandwidth Needed

Relative Permittivity of LTCC

(Low Temperature Co-fired Ceramic)

PC Board Height for LTCC

Transmission Line Synthesis Function

$$W(\epsilon_r, h, Z_0) := \begin{cases} A \leftarrow \frac{Z_0}{60\text{ohm}} \cdot \sqrt{\frac{\epsilon_r + 1}{2}} + \frac{\epsilon_r - 1}{\epsilon_r + 1} \cdot \left(0.23 + \frac{0.11}{\epsilon_r} \right) \\ B \leftarrow \frac{377\text{ohm} \cdot \pi}{2 \cdot Z_0 \cdot \sqrt{\epsilon_r}} \\ W_1 \leftarrow \frac{8 \cdot e^A \cdot h}{e^{2 \cdot A} - 2} \\ W_2 \leftarrow \frac{2 \cdot h}{\pi} \left[B - 1 - \ln(2 \cdot B - 1) + \frac{\epsilon_r - 1}{2 \cdot \epsilon_r} \cdot \left(\ln(B - 1) + 0.39 - \frac{0.61}{\epsilon_r} \right) \right] \\ W \leftarrow \text{if} \left(\frac{W_1}{h} < 2, W_1, \text{if} \left(\frac{W_2}{h} > 2, W_2, 0\text{m} \right) \right) \\ W \end{cases}$$

Single Shunt Stub Transmission Line Matching

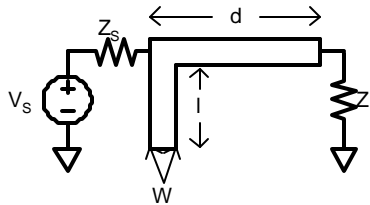


Fig. 1: Single open shunt stub matching

Here a stub and a transmission line are used to match a transmission line input, with characteristic impedance of Z_S , to a complex load of impedance Z_L . A open circuit stub, of length l , is placed at the input with a transmission line connecting it to the load with a length, d . Both transmission line segments have a characteristic impedance of Z_S . You little (choice of two implementations) control over the bandwidth for single stub tuners.

$$W_{\text{val}} := W(\epsilon_r, h, Z_S)$$

$$W_{\text{val}} = 3.016\text{mm}$$

Width of Transmission Lines

$$\omega_c := 2 \cdot \pi \cdot f_c$$

$$\omega_c = 6.283 \times 10^9 \frac{\text{rad}}{\text{sec}}$$

Center Frequency

$$\lambda := \frac{2 \cdot \pi \cdot c}{\omega_c}$$

$$\lambda = 29.9\text{cm}$$

Wavelength in Free Space

$$R_L := \text{Re}(Z_L)$$

$$R_L = 100\Omega$$

Real Part of Load Impedance

$$X_L := \text{Im}(Z_L)$$

$$X_L = 0\Omega$$

Imag part of Load Impedance

When designing transmission line transformers, there are multiple solutions to the matching problem. For an example, an additional wavelength of transmission line can be added to yield the same matching, but with a different frequency response. This is usually not done to keep the area small. Here multiple solutions are presented.

$$t_1 := \frac{X_L + \sqrt{\frac{R_L}{Z_S} \cdot [(Z_S - R_L)^2 + X_L^2]}}{R_L - Z_S}$$

$$t_1 = 1.414$$

Coefficient for First Solution

$$t_2 := \frac{X_L - \sqrt{\frac{R_L}{Z_S} \cdot [(Z_S - R_L)^2 + X_L^2]}}{R_L - Z_S}$$

$$t_2 = -1.414$$

Coefficient for Second Solution

$$d_1 := \lambda \cdot \frac{1}{2 \cdot \pi} \cdot \text{atan}(t_1)$$

$$d_1 = 4.546 \text{ cm}$$

$$d_2 := \lambda \cdot \frac{1}{2 \cdot \pi} \cdot (\pi + \text{atan}(t_2))$$

$$d_2 = 10.404 \text{ cm}$$

$$B_{s1} := -\frac{R_L^2 \cdot t_1 - (Z_S - X_L \cdot t_1) \cdot (X_L + Z_S \cdot t_1)}{Z_S \left[R_L^2 + (X_L + Z_S \cdot t_1)^2 \right]}$$

$$\frac{1}{B_{s1}} = -70.711 \text{ ohm}$$

$$B_{s2} := -\frac{R_L^2 \cdot t_2 - (Z_S - X_L \cdot t_2) \cdot (X_L + Z_S \cdot t_2)}{Z_S \left[R_L^2 + (X_L + Z_S \cdot t_2)^2 \right]}$$

Connecting Transmission L (Solution #1&3)

Connecting Transmission L (Solution #2&4)

Susceptance of Stub (Solution #1&3)

Susceptance of Stub (Solution #2&4)

$$\frac{1}{B_{s2}} = 70.711 \text{ ohm}$$

$$l_{o1} := \frac{\lambda}{2 \cdot \pi} \cdot \text{atan}(B_{s1} \cdot Z_S)$$

$$l_{o1} = -29.289 \text{ mm}$$

$$l_{s1} := \frac{-\lambda}{2 \cdot \pi} \cdot \text{atan}\left(\frac{1}{B_{s1} \cdot Z_S}\right)$$

$$l_{s1} = 45.461 \text{ mm}$$

$$l_{o2} := \frac{\lambda}{2 \cdot \pi} \cdot \text{atan}(B_{s2} \cdot Z_S)$$

$$l_{o2} = 29.289 \text{ mm}$$

$$l_{s2} := \frac{-\lambda}{2 \cdot \pi} \cdot \text{atan}\left(\frac{1}{B_{s2} \cdot Z_S}\right)$$

$$l_{s2} = -45.461 \text{ mm}$$

Length of Open Circuit Stub (Solution #1)

Length of Short Circuit Stub (Solution #3)

Length of Open Circuit Stub (Solution #2)

Length of Short Circuit Stub (Solution #4)

Some solutions are not valid (negative lengths), so they are dropped:

$$OK_1 := (l_{o1} > 0) \cdot (d_1 > 0)$$

$$OK_1 = 0$$

$$OK_2 := (l_{o2} > 0) \cdot (d_2 > 0)$$

$$OK_2 = 1$$

$$OK_3 := (l_{s1} > 0) \cdot (d_1 > 0)$$

$$OK_3 = 1$$

$$OK_4 := (l_{s2} > 0) \cdot (d_2 > 0)$$

$$OK_4 = 0$$

$$\text{Area}_1 := (l_{o1} + d_1) \cdot W_{\text{val}}$$

$$\text{Area}_1 = 0.488 \text{ cm}^2$$

$$\text{Area}_2 := (l_{o2} + d_2) \cdot W_{\text{val}}$$

$$\text{Area}_2 = 4.021 \text{ cm}^2$$

$$\text{Area}_3 := (l_{s1} + d_1) \cdot W_{\text{val}}$$

$$\text{Area}_3 = 2.742 \text{ cm}^2$$

$$\text{Area}_4 := (l_{s2} + d_2) \cdot W_{\text{val}}$$

$$\text{Area}_4 = 1.767 \text{ cm}^2$$

Is Solution #1 OK to use?

Is Solution #2 OK to use?

Is Solution #3 OK to use?

Is Solution #4 OK to use?

Area for Solution #1

Area for Solution #2

Area for Solution #1

Area for Solution #2

$$l_{o1} \cdot d_1 = -13.315 \text{ cm}^2$$

Areas if white space is considered

$$l_{o2} \cdot d_2 = 30.472 \text{ cm}^2$$

$$l_{s1} \cdot d_1 = 20.667 \text{ cm}^2$$

$$l_{s2} \cdot d_2 = -47.297 \text{ cm}^2$$

Single Series Stub Transmission Line Matching

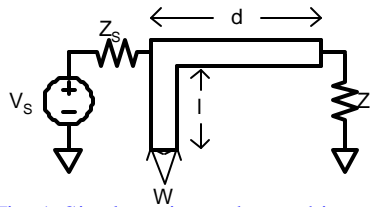


Fig. 1: Single series stub matching

$$G_L := \operatorname{Re}\left(\frac{1}{Z_L}\right) \quad B_L := \operatorname{Im}\left(\frac{1}{Z_L}\right)$$

Load Conductance and Suseptance

$$Y_S := \frac{1}{Z_S}$$

Source Admittance

$$t_1 := \frac{B_L + \sqrt{\frac{G_L}{Y_S} \cdot [(Y_S - G_L)^2 + B_L^2]}}{G_L - Y_S}$$

Coefficient for First Solution

$$t_2 := \frac{B_L - \sqrt{\frac{G_L}{Y_S} \cdot [(Y_S - G_L)^2 + B_L^2]}}{G_L - Y_S}$$

Coefficient for Second Solution

$$d_1 := \lambda \cdot \frac{1}{2 \cdot \pi} \cdot \operatorname{atan}(t_1)$$

$$d_1 = -2.929 \text{ cm}$$

Connecting Transmission Line (Solution #1&3)

$$d_2 := \lambda \cdot \frac{1}{2 \cdot \pi} \cdot (\pi + \operatorname{atan}(t_2))$$

$$d_2 = 17.879 \text{ cm}$$

Connecting Transmission Line (Solution #2&4)

$$X_{s1} := \frac{G_L^2 \cdot t_1 - (Y_S - t_1 \cdot B_L) \cdot (B_L + t_1 \cdot Y_S)}{-Y_S [G_L^2 + (B_L + t_1 \cdot Y_S)^2]}$$

$$X_{s1} = -35.355 \Omega \text{ Stub Reactance (Solution \#1)}$$

$$X_{s2} := \frac{G_L^2 \cdot t_2 - (Y_S - t_2 \cdot B_L) \cdot (B_L + t_2 \cdot Y_S)}{-Y_S [G_L^2 + (B_L + t_2 \cdot Y_S)^2]}$$

$$X_{s2} = 35.355 \Omega \text{ Stub Reactance (Solution \#2)}$$

$$l_{o1} := \frac{-\lambda}{2 \cdot \pi} \cdot \operatorname{atan}\left(\frac{Z_S}{X_{s1}}\right)$$

$$l_{o1} = 45.461 \text{ mm}$$

Length of Open Circuit Stub (Solution #1)

$$l_{s1} := \frac{\lambda}{2 \cdot \pi} \cdot \operatorname{atan}\left(\frac{X_{s1}}{Z_S}\right)$$

$$l_{s1} = -29.289 \text{ mm}$$

Length of Short Circuit Stub (Solution #3)

$$l_{o2} := \frac{-\lambda}{2 \cdot \pi} \cdot \operatorname{atan}\left(\frac{Z_S}{X_{s2}}\right)$$

$$l_{o2} = -45.461 \text{ mm}$$

Length of Open Circuit Stub (Solution #2)

$$l_{s2} := \frac{\lambda}{2 \cdot \pi} \cdot \operatorname{atan}\left(\frac{X_{s2}}{Z_S}\right)$$

$$l_{s2} = 29.289 \text{ mm}$$

Length of Short Circuit Stub (Solution #4)

$$OK_1 := (l_{o1} > 0) \cdot (d_1 > 0)$$

$$OK_1 = 0$$

Is Solution #1 OK to use?

$$OK_2 := (l_{o2} > 0) \cdot (d_2 > 0)$$

$$OK_2 = 0$$

Is Solution #2 OK to use?

$$OK_3 := (l_{s1} > 0) \cdot (d_1 > 0)$$

$$OK_3 = 0$$

Is Solution #3 OK to use?

$$OK_4 := (l_{s2} > 0) \cdot (d_2 > 0)$$

$$OK_4 = 1$$

Is Solution #4 OK to use?

$$\text{Area}_1 := (l_{o1} + d_1) \cdot W_{\text{val}}$$

$$\text{Area}_1 = 0.488 \text{ cm}^2$$

Area for Solution #1

$$\text{Area}_2 := (l_{o2} + d_2) \cdot W_{\text{val}}$$

$$\text{Area}_2 = 4.021 \text{ cm}^2$$

Area for Solution #2

$$\text{Area}_3 := (l_{s1} + d_1) \cdot W_{\text{val}}$$

$$\text{Area}_3 = -1.767 \text{ cm}^2$$

Area for Solution #1

$$\text{Area}_4 := (l_{s2} + d_2) \cdot W_{\text{val}}$$

$$\text{Area}_4 = 6.276 \text{ cm}^2$$

Area for Solution #2

1/4 Wave Transformer Matching

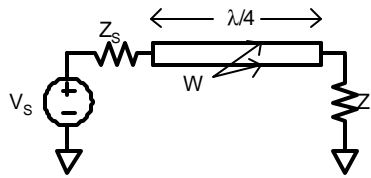


Fig. 1: Single shunt stub matching

No control over bandwidth achieved

$$\Delta f_{fc} := \frac{\Delta f}{f_c}$$

$$\Delta f_{fc} = 30\%$$

Fractional Bandwidth Needed

$$\Gamma_{max} := 10^{10 \cdot \frac{S_{11max}}{20}}$$

$$\Gamma_{max} = 0.1$$

Reflection Coefficient Needed

$$Z_{1_4} := \sqrt{Z_S \cdot Z_L}$$

$$Z_{1_4} = 70.711 \Omega$$

Impedance of Quarter Wave Transm

$$\Delta f_{f_{cmax}} := 2 - \frac{4}{\pi} \cdot \arccos \left(\frac{\Gamma_{max}}{\sqrt{1 - \Gamma_{max}^2}} \cdot \frac{2 \cdot \sqrt{Z_S \cdot Z_L}}{|Z_L - Z_S|} \right)$$

Actual Bandwidth

$$\Delta f_{f_{cmax}} = 36.7\%$$

$$OK := \Delta f_{f_{cmax}} > \Delta f_{fc}$$

$$OK = 1$$

OK to use this circuit?

$$\beta := \frac{2 \cdot \pi}{\lambda} \cdot \sqrt{\epsilon_r}$$

$$\beta = 43.066 \text{ m}^{-1}$$

Phase Constant (Wave Number)

$$\text{len} := \frac{\lambda}{4}$$

$$\text{len} = 7.475 \text{ cm}$$

Length of Transformer

$$W_{val} := W(\epsilon_r, h, Z_{1_4})$$

$$W_{val} = 1.61 \text{ mm}$$

Width of Transformer

$$\text{Area} := \text{len} \cdot W_{val}$$

$$\text{Area} = 1.203 \text{ cm}^2$$

Area of Transformer

Outputs

$$Z_{1_4} = 70.711 \Omega$$

Impedance of Line

$$OK = 1$$

OK to use this circuit?

$$\text{Area} = 1.203 \text{ cm}^2$$

Area of Transformer

1/4 Wave Binomial Transformer Matching

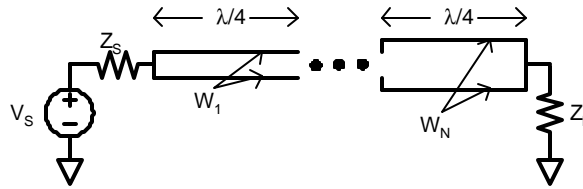


Fig. 1: 1/4 wave binomial transformer matching

$$\Gamma_{DC} := \frac{Z_L - Z_S}{Z_L + Z_S}$$

$$\Gamma_{DC} = 0.333$$

DC Reflection Coefficient

$$\frac{\ln\left(\frac{\Gamma_{max}}{|\Gamma_{DC}|}\right)}{\ln\left[\cos\left[\frac{\pi}{2} \cdot \left(1 - \frac{\Delta f_{fc}}{2}\right)\right]\right]} = 0.828$$

Fractional Order Needed

$$N := \text{ceil}\left[\frac{\ln\left(\frac{\Gamma_{max}}{|\Gamma_{DC}|}\right)}{\ln\left[\cos\left[\frac{\pi}{2} \cdot \left(1 - \frac{\Delta f_{fc}}{2}\right)\right]\right]}\right]$$

$$N = 1$$

Order Estimation

$$A(N) := 2^{-N} \cdot \Gamma_{DC}$$

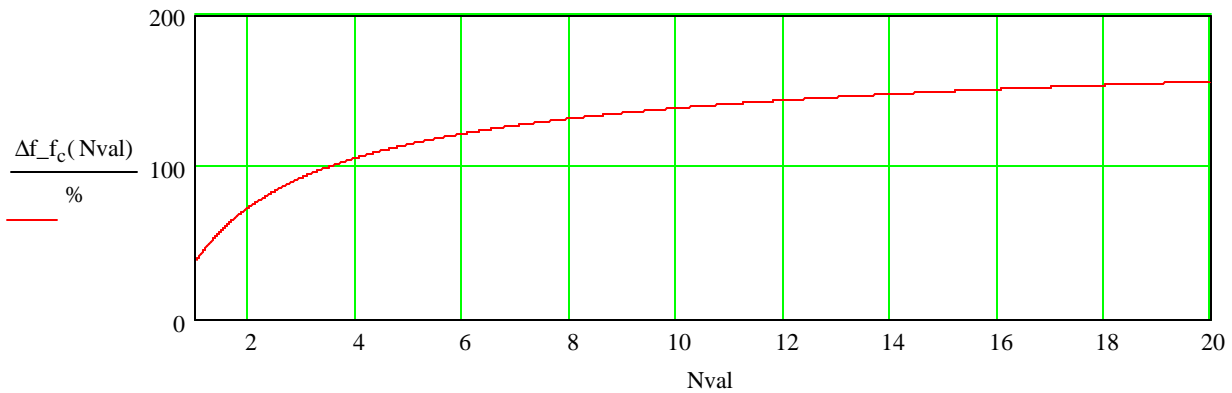
$$A(N) = 0.167$$

Binomial Coefficient

$$\Delta f_{fc}(N) := 2 - \frac{4}{\pi} \cdot \text{acos}\left[\frac{1}{2} \cdot \left(\frac{\Gamma_{max}}{|A(N)|}\right)^{\frac{1}{N}}\right]$$

$$\Delta f_{fc}(N) = 38.795\%$$

Actual Fractional Bandwidth



$$i := 0..(N - 1)$$

Index Vector

$$C(n, N) := \frac{N!}{(N - n)! \cdot n!}$$

$$C(i, N) =$$

Binomial Coefficients

$$\boxed{1}$$

$$\Gamma(n, N) := A(N) \cdot C(n, N)$$

$$\Gamma(i, N) =$$

Reflection Coefficients

$$\boxed{0.167}$$

$j := 1..N$

$Z := \begin{cases} Z_{val_0} \leftarrow Z_S \\ \text{for } i \in 0..(N-1) \\ \quad Z_{val_{i+1}} \leftarrow Z_{val_i} \cdot \left(\frac{\Gamma(i, N) + 1}{1 - \Gamma(i, N)} \right) \\ Z_{val} \end{cases}$

$W_{val_j} := W(\epsilon_r, h, Z_j)$

$\max(W_{val}) = 1.643 \text{ mm}$

$len := \frac{\lambda}{4} \cdot N$

$Area := len \cdot \max(W_{val})$

Outputs

$W_{val} = \begin{pmatrix} 0 \\ 1.643 \end{pmatrix} \text{ mm}$

$len = 7.475 \text{ cm}$

$Area = 1.228 \text{ cm}^2$

$Z_j =$
 $70 \ \Omega$

T-Line Impedances

$W_{val} = \begin{pmatrix} 0 \\ 1.643 \end{pmatrix} \text{ mm}$

Widths of Transmission Line Segme

Maximum Width

$len = 7.475 \text{ cm}$

Length of Transformer

$Area = 1.228 \text{ cm}^2$

Area of Transformer

Widths of Line Segments

Length of Transformer

Area of Transformer

1/4 Wave Chebyshev Transformer Matching

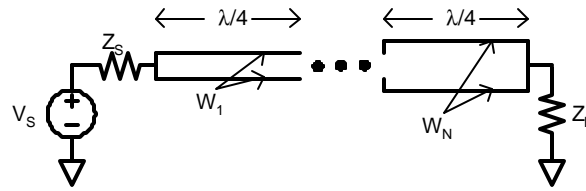


Fig. 1: 1/4 wave chebyshev transformer matching

$$\frac{\operatorname{acosh}\left(\frac{|\Gamma_{DC}|}{\Gamma_{\max}}\right)}{\operatorname{acosh}\left[\sec\left[\frac{\pi}{2}\left(1 - \frac{\Delta f}{2 \cdot f_c}\right)\right]\right]} = 0.878$$

Fractional Order Needed

$$N := \operatorname{ceil}\left[\frac{\operatorname{acosh}\left(\frac{|\Gamma_{DC}|}{\Gamma_{\max}}\right)}{\operatorname{acosh}\left[\sec\left[\frac{\pi}{2}\left(1 - \frac{\Delta f}{2 \cdot f_c}\right)\right]\right]}\right]$$

$$N = 1$$

Order Estimation

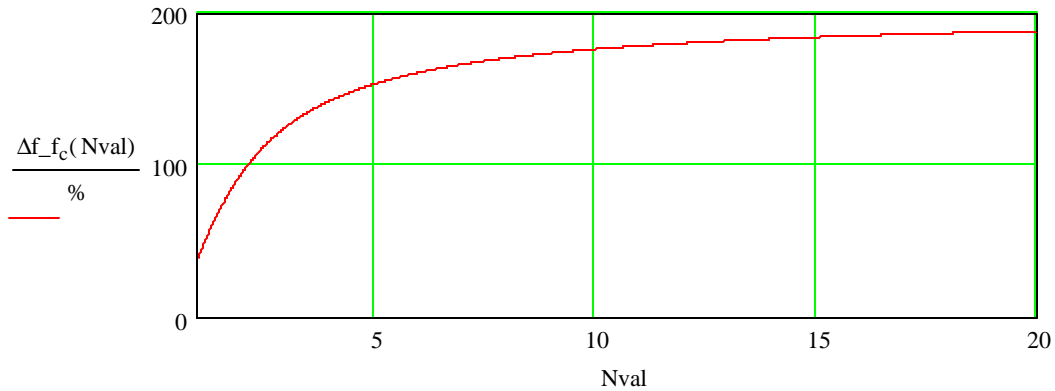
$$N := \operatorname{if}(N > 2, N, 2)$$

$$N = 2$$

$$\Delta f_{f_c}(N) := 2 - \frac{4}{\pi} \cdot \operatorname{asec}\left(\cosh\left(\frac{1}{N} \cdot \operatorname{acosh}\left(\frac{|\Gamma_{DC}|}{\Gamma_{\max}}\right)\right)\right)$$

$$\Delta f_{f_c}(N) = 95.098 \%$$

Actual Fractional Bandwidth



$$A := \Gamma_{\max}$$

$$A = 0.1$$

Transformer Coefficient

$$\theta_{\max} := \operatorname{asec}\left(\cosh\left(\frac{1}{N} \cdot \operatorname{acosh}\left(\frac{\ln\left(\frac{Z_L}{Z_S}\right)}{2 \cdot \Gamma_{\max}}\right)\right)\right)$$

$$\theta_{\max} = 47.993 \text{ deg}$$

$T(n, x) := \text{if}(|x| < 1, \cos(n \cdot \arccos(x)), \cosh(n \cdot \operatorname{acosh}(x)))$
 $i := 0..N$

Chebyshev Polynomial Coefficients
 Index Vector

$$\Gamma(n) := \begin{cases} \frac{A}{2} \cdot \sec(\theta_{\max})^3 & \text{if } n = 0 \\ \frac{3}{2} \cdot A \cdot \left(\sec(\theta_{\max})^3 - \sec(\theta_{\max}) \right) & \text{if } n = 1 \\ \frac{3}{2} \cdot A \cdot \left(\sec(\theta_{\max})^3 - \sec(\theta_{\max}) \right) & \text{if } n = 2 \\ \frac{A}{2} \cdot \sec(\theta_{\max})^3 & \text{if } n = 3 \end{cases}$$

Reflection Coefficients

$\Gamma(i) =$

0.167
0.276
0.276

$j := 1..N$

$$Z := \begin{cases} Z_{\text{val}_0} \leftarrow Z_S \\ \text{for } i \in 0..N-1 \\ Z_{\text{val}_{i+1}} \leftarrow Z_{\text{val}_i} \cdot e^{2 \cdot \Gamma(i)} \\ Z_{\text{val}} \end{cases}$$

$Z_j =$ Transmission Line Impedances

69.803	Ω
121.31	

$W_{\text{val}_j} := W(\epsilon_r, h, Z_j)$

$W_{\text{val}} = \begin{pmatrix} 0 \\ 1.652 \\ 0.401 \end{pmatrix} \text{ mm}$ Widths of Transmission Line Segments

$\max(W_{\text{val}}) = 1.652 \text{ mm}$

Maximum Width

$\text{len} := \frac{\lambda}{4} \cdot N$

$\text{len} = 14.95 \text{ cm}$ Length of Transformer

$\text{Area} := \text{len} \cdot \max(W_{\text{val}})$

$\text{Area} = 2.47 \text{ cm}^2$ Area of Transformer

Outputs

$W_{\text{val}} = \begin{pmatrix} 0 \\ 1.652 \\ 0.401 \end{pmatrix} \text{ mm}$
$\text{len} = 14.95 \text{ cm}$
$\text{Area} = 2.47 \text{ cm}^2$

Widths of Transmission Line Segments
 Length of Transformer
 Area of Transformer

Exponential Taper Matching Transformer

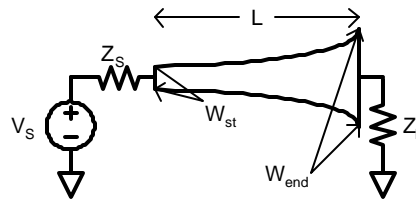


Fig. 1: Exponential transformer matching

$$L := \frac{\pi}{\beta}$$

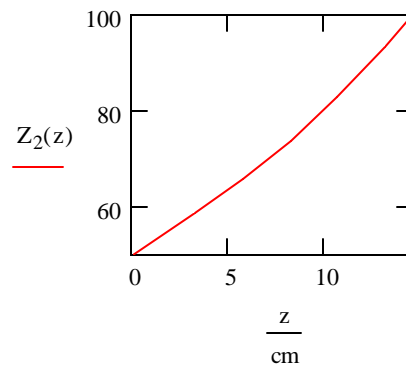
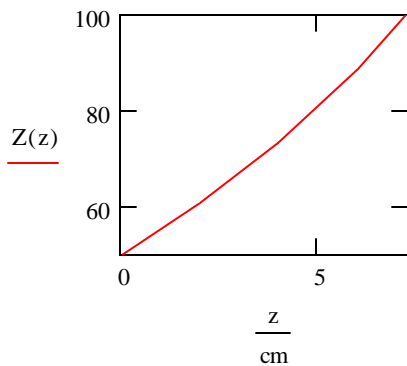
$L = 7.295 \text{ cm}$ Length of Transformer (Option #1: $\beta L = \pi$)

$$L_2 := \frac{2 \cdot \pi}{\beta}$$

$L_2 = 14.59 \text{ cm}$ Length of Transformer (Option #2: $\beta L = 2\pi$)

$$Z(z) := Z_S \cdot e^{\frac{1}{L} \cdot \ln\left(\frac{Z_L}{Z_S}\right) \cdot z}$$

$$Z_2(z) := Z_S \cdot e^{\frac{1}{L_2} \cdot \ln\left(\frac{Z_L}{Z_S}\right) \cdot z}$$



$$W(\epsilon_r, h, Z(0\text{cm})) = 3.016 \text{ mm}$$

Width at Beginning of Taper

$$W(\epsilon_r, h, Z(L)) = 0.714 \text{ mm}$$

Width at End of Taper

$$\text{TotArea} := L \cdot W(\epsilon_r, h, Z(0\text{cm}))$$

$$\text{TotArea} = 2.2 \text{ cm}^2$$

Total Area of Transformer

Triangular Taper Matching Transformer

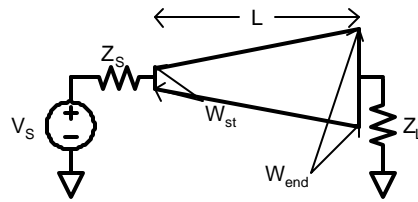


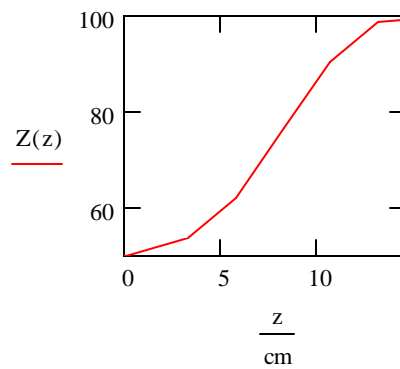
Fig. 1: Triangular transformer matching

$$L := \frac{2 \cdot \pi}{\beta}$$

$$L = 14.59 \text{ cm}$$

Length of Transformer (Option #1)

$$Z(z) := \text{if} \left[z < \frac{L}{2}, Z_S \cdot e^{2 \cdot \left(\frac{z}{L}\right)^2 \cdot \ln\left(\frac{Z_L}{Z_S}\right)}, Z_S \cdot e^{\left[4 \cdot \frac{z}{L} - 2 \cdot \left(\frac{z}{L}\right)^2 - 1\right] \cdot \ln\left(\frac{Z_L}{Z_S}\right)} \right]$$



$$W(\epsilon_r, h, Z(0 \text{ cm})) = 3.016 \text{ mm}$$

$$W(\epsilon_r, h, Z(L)) = 0.714 \text{ mm}$$

$$\text{TotArea} := L \cdot W(\epsilon_r, h, Z(0 \text{ cm}))$$

$$\text{TotArea} = 4.401 \text{ cm}^2$$

Width at Beginning of Taper

Width at End of Taper

Total Area of Transformer

Klopfenstein Taper Matching Transformer

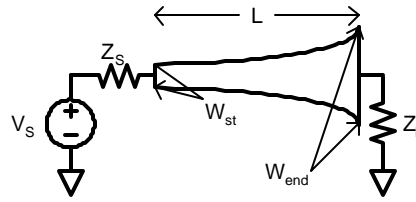


Fig. 1: Klopfenstein transformer matching

$$A := \operatorname{acosh}\left(\frac{\Gamma_{\text{DC}}}{\Gamma_{\text{max}}}\right)$$

$$A = 1.874$$

Coefficient for Transformer

$$L := \frac{A}{\beta}$$

$$L = 4.351 \text{ cm}$$

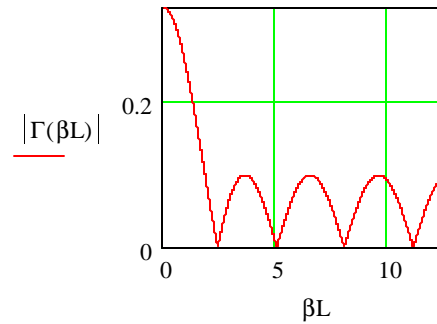
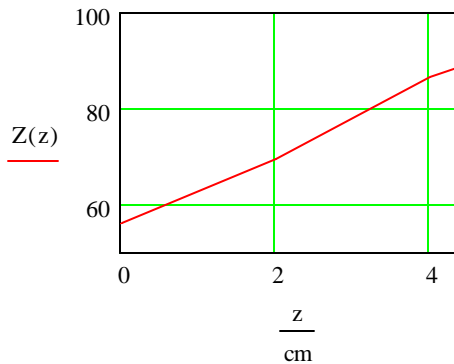
Length of Transformer

$$\phi(x, A) := \int_0^x \frac{\operatorname{Ei}\left(A \cdot \sqrt{1-y^2}\right)}{A \cdot \sqrt{1-y^2}} dy$$

$$Z(z) := Z_S \cdot e^{\frac{1}{2} \cdot \ln\left(\frac{Z_L}{Z_S}\right) + \frac{\Gamma_{\text{DC}}}{\cosh(A)} \cdot A^2 \cdot \phi\left(\frac{2 \cdot z}{L} - 1, A\right)}$$

$$Z(0 \text{ cm}) = 55.995 \Omega \quad Z(L) = 89.294 \Omega$$

$$\Gamma(\beta L) := \Gamma_{\text{DC}} \cdot e^{-\sqrt{-1} \cdot \beta L} \cdot \frac{\cos\left(\sqrt{\beta L^2 - A^2}\right)}{\cosh(A)}$$



$$W(\epsilon_r, h, Z(0 \text{ cm})) = 2.492 \text{ mm}$$

Width at Beginning of Taper

$$W(\epsilon_r, h, Z(L)) = 0.956 \text{ mm}$$

Width at End of Taper

$$\text{TotArea} := L \cdot W(\epsilon_r, h, Z(0 \text{ cm}))$$

$$\text{TotArea} = 1.084 \text{ cm}^2$$

Total Area of Transformer

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