



General Smith Chart Matching

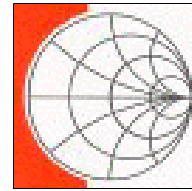


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General Impedance Matching

This worksheets discusses in detail graphical impedance matching with a Smith chart. A few notes on matching networks before we start: Matching networks have two main purposes. The first is to deliver allow the maximum power transfer from a source in series with a source impedance of Z_S to a load impedance Z_L . The second is to provide a desired impedance to a circuit to achieve other desired characteristics, such as to filter, to maintain a desired filter transfer characteristics, to an LNA to provide a low noise figure, or to a transmission line to reduce reflections. Here the matching networks are designed for narrowband networks, such as RF transmitters and receivers, but not broadband applications, such as audio applications, cable modems, backplanes, ethernet, etc.

Let's start with the available source power and load power definitions:

$$P_S = \frac{V_{S_{rms}}^2}{R_S} \quad P_L = \frac{V_{L_{rms}}^2}{R_L} \quad V_{L_{rms}} = V_{S_{rms}} \cdot \frac{Z_L}{Z_S + Z_L}$$

The power gain of the network is a found by dividing the expression for the output power divided by the input power.

$$G_p = \frac{P_L}{P_S} = \left(\frac{Z_L}{Z_S + Z_L} \right)^2 \cdot \frac{R_S}{R_L} = \left[\frac{R_L^2 + X_L^2}{(R_S + R_L)^2 + (X_L + X_S)^2} \right] \cdot \frac{R_S}{R_L}$$

To optimize the power gain for a given load impedance the derivative of power gain with respect to the source resistance and reactance is set to zero, and the source impedance is solved. This is easiest done in a two step process by first finding the optimal source reactance, $\frac{d}{dX_S} G_p = 0$, which gives an optimal source reactance of:

$$X_{S_{opt}} = -X_L$$

The source resistance can be found similarly with $\frac{d}{dR_S} G_p = 0$, giving an optimal source resistance of:

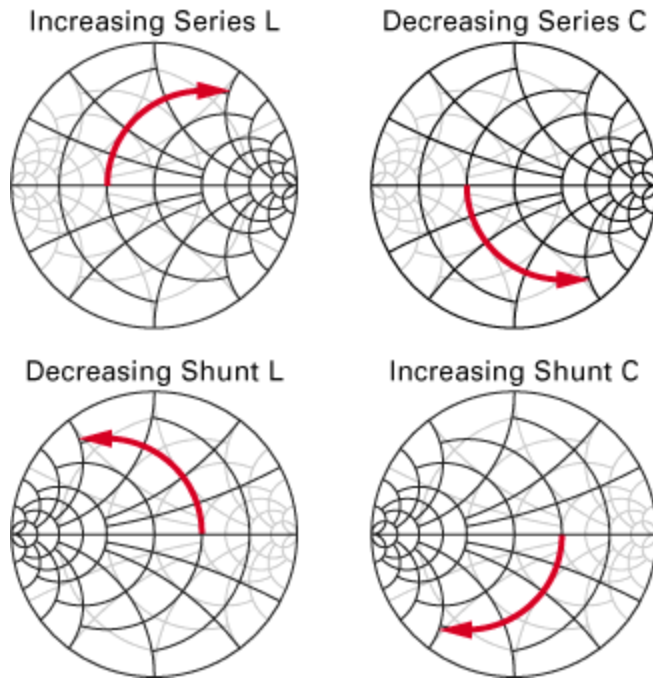
$$R_{S_{opt}} = R_L$$

This impedance is also said to give zero power reflection. That is all of the power is transferred to the load, and none is reflected back to source. When viewing the power signals as waves it is difficult to view a reflection, when the matching network consists of discrete elements. When studying distributed structures, such as transmission lines, we see the find that zero reflection occurs for true matched line and load impedance (not conjugate). So if you are delivering power from a source through an input matching network, through a transmission line, through an output matching network, and finally to a load, how is that you can provide maximum power delivery to the load with a conjugate match, but have zero reflections? The answer is: "You can't, " because this situation does not occur in practice. In practice in order for a transmission line to have an imaginary characteristic impedance it must be lossy.

Impedance Transformation for Power Amplifiers

Careful! Output networks for power amplifiers are more commonly "impedance transformation" networks rather than "impedance matching" networks. The goal of a power amplifier is to efficiently transfer power from a **DC** power supply to a load at an RF frequency; however an impedance matching network provides maximum power transfer from an **RF** source to a load. In turns out the two are the same for output powers less than $(V_{DD} - V_{DSsat})^2 / 2 / R_L$, which is around 100mW for a 50 ohm load and 3V supply. For powers above this, the load is **transformed** to an impedance, seen from the power amplifier, which is much lower than the output impedance of the PA.

Impedance Matching with a Smith Chart



I am personally not a big fan of smith charts, but they have some historical value, are commonly used in network analyzers, and they allow you to plot infinite change in impedance in a finite area. Here we describe a common older method for graphically matching a complex source impedance to a complex load impedance is using a Smith chart.

- ▢ useful functions and identities
- ▢ Units
- ▢ Constants

Inputs

$$Z_L := 50\text{ohm} + j \cdot 30\text{ohm}$$

Load impedance

$$Z_S := 50\text{ohm} + j \cdot 0.01\text{ohm}$$

Source impedance

$$f := 900\text{MHz}$$

Frequency

$$\omega := 2 \cdot \pi \cdot f$$

$$Z_0 := 50\Omega$$

Reference impedance for S parameters

Step 1:

The source and load impedances are converted to reflection coefficients and placed on the smith chart. One of the two impedances is conjugated, so that a conjugate match is achieved. Here we arbitrarily choose the source to be the conjugated impedance.

$$\Gamma_S := \frac{Z_S - Z_0}{Z_S + Z_0}$$

$$|\Gamma_S| = 10 \times 10^{-5} \quad \angle \Gamma_S = 89.994^\circ$$

$$\Gamma_L := \frac{\overline{Z_L} - Z_0}{\overline{Z_L} + Z_0}$$

$$|\Gamma_L| = 0.863$$

$$\angle \Gamma_L = -117.699^\circ$$

Step 2-: Starting with a source impedance we can gradually add series or parallel elements to provide the necessary match and frequency response. Using the following syntax:

- sp:** *series or parallel:* **s**=series, **p**=parallel (shunt)
- LCtype:** *element type:* **R**=resistance, **C**=capacitance, **L**=inductance, **T**=transmission line
- LC:** *element value:* unitless value for ohms, farads, henries, or characteristic impedance in ohms.
- g*l:** *propagation constant:* if a transmission line
- Z_{end}:** *end impedance of a shunt transmission line:* usually zero or infinity.

In the following example we use all the possible elements for illustration

Network Elements

Shunt Inductors:

For parallel inductors, a segment is plotted, which starts with the previous impedance and sweeps the inductance down from infinity to the specified value.

$$\text{LCtype}_1 := "L" \quad \text{sp}_1 := "p" \quad \text{LC}_1 := \frac{10\text{nH}}{\text{H}}$$



Series Capacitors:

For series capacitors, a segment is plotted, which starts with the previous impedance and sweeps the capacitance from 0 to the specified value.

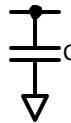
$$\text{LCtype}_2 := "C" \quad \text{sp}_2 := "s" \quad \text{LC}_2 := \frac{2\text{pF}}{\text{F}}$$



Shunt Capacitors:

For parallel capacitors, a segment is plotted, which starts with the previous impedance and sweeps the capacitance from 0 to the specified value.

$$\text{LCtype}_3 := "C" \quad \text{sp}_3 := "p" \quad \text{LC}_3 := \frac{1\text{pF}}{\text{F}}$$



Series Resistors:

For series resistors, a segment is plotted, which starts with the previous impedance and sweeps the resistance from 0 to the specified value.

$$\text{LCtype}_4 := "R" \quad \text{sp}_4 := "s" \quad \text{LC}_4 := \frac{0.01\Omega}{\Omega}$$



Shunt Resistors:

For parallel resistors, a segment is plotted, which starts with the previous impedance and sweeps the resistance from infinity to the specified value.

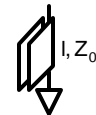
$$\text{LCtype}_5 := "R" \quad \text{sp}_5 := "p" \quad \text{LC}_5 := \frac{10^6\Omega}{\Omega}$$



Shorted Shunt Transmission Lines:

For shorted shunt transmission lines (which look like shunt inductors), a segment is plotted, which starts with the previous impedance and sweeps in the electrical length ($j*\gamma*\text{len}$) from $\pi/2$ (which makes the line look like an open) down to the specified value.

$$\begin{array}{l} \text{R}_1 := 0 \frac{\text{ohm}}{\text{cm}} \quad \text{G}_1 := \frac{0}{\text{ohm}\cdot\text{cm}} \quad \text{C}_1 := 2 \frac{\text{pF}}{\text{cm}} \quad \text{L}_1 := \frac{0.5\text{nH}}{\text{mm}} \quad \text{Z}_{\text{end}_6} := 0\Omega \\ \text{LCtype}_6 := "T" \quad \text{sp}_6 := "p" \quad \text{LC}_6 := \frac{\sqrt{\frac{\text{R}_1 + j\omega \cdot \text{L}_1}{\text{G}_1 + j\omega \cdot \text{C}_1}}}{\Omega} \quad \text{len} := 27.8\text{mm} \quad \text{LC}_6 = 50 \end{array}$$



$$\gamma_6 := \sqrt{(R_1 + j \cdot \omega \cdot L_1) \cdot (G_1 + j \cdot \omega \cdot C_1)} \cdot \text{len}$$

$$\gamma_6 = 1.572i$$

Open Shunt Transmission Line:

For open shunt transmission lines (which look like shunt capacitors), a segment is plotted, which starts with the previous impedance and sweeps in the length from 0 to the specified value.

$$R_1 := 0 \frac{\text{ohm}}{\text{cm}}$$

$$G_1 := \frac{0}{\text{ohm} \cdot \text{cm}}$$

$$C_1 := 2 \frac{\text{pF}}{\text{cm}}$$

$$L_1 := \frac{0.5 \text{nH}}{\text{mm}}$$

$$Z_{\text{end}_7} := 10^6 \Omega$$



$$\text{LCtype}_7 := "T"$$

$$\text{sp}_7 := "p"$$

$$\text{LC}_7 := \sqrt{\frac{R_1 + j \cdot \omega \cdot L_1}{G_1 + j \cdot \omega \cdot C_1}} \frac{1}{\Omega}$$

$$\text{len} := 2 \mu\text{m}$$

$$\text{LC}_7 = 50$$

$$\gamma_7 := \sqrt{(R_1 + j \cdot \omega \cdot L_1) \cdot (G_1 + j \cdot \omega \cdot C_1)} \cdot \text{len}$$

$$\gamma_7 = 1.131i \times 10^{-4}$$

Series Transmission Line:

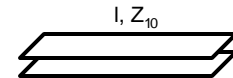
For series transmission lines, a segment is plotted, which starts with the previous impedance and sweeps in the length from 0 to the specified value.

$$R_1 := 0 \frac{\text{ohm}}{\text{cm}}$$

$$G_1 := \frac{0}{\text{ohm} \cdot \text{cm}}$$

$$C_1 := 2 \frac{\text{pF}}{\text{cm}}$$

$$L_1 := \frac{0.5 \text{nH}}{\text{mm}}$$



$$\text{LCtype}_8 := "T"$$

$$\text{sp}_8 := "s"$$

$$\text{LC}_8 := \sqrt{\frac{R_1 + j \cdot \omega \cdot L_1}{G_1 + j \cdot \omega \cdot C_1}} \frac{1}{\Omega}$$

$$\text{len} := 4 \text{mm}$$

$$\text{LC}_8 = 50$$

$$\gamma_8 := \sqrt{(R_1 + j \cdot \omega \cdot L_1) \cdot (G_1 + j \cdot \omega \cdot C_1)} \cdot \text{len}$$

$$\gamma_8 = 0.226i$$

Series Inductors:

For series inductors, a segment is plotted, which starts with the previous impedance and sweeps the inductance from 0 to the specified value.

$$\text{LCtype}_9 := "L"$$

$$\text{sp}_9 := "s"$$

$$\text{LC}_9 := 0.1 \frac{\text{nH}}{\text{H}}$$



Overall Circuit Schematic

As you add or remove circuit elements above the following picture will update to reflect the actual schematic.

Images

Q and VSWR contours

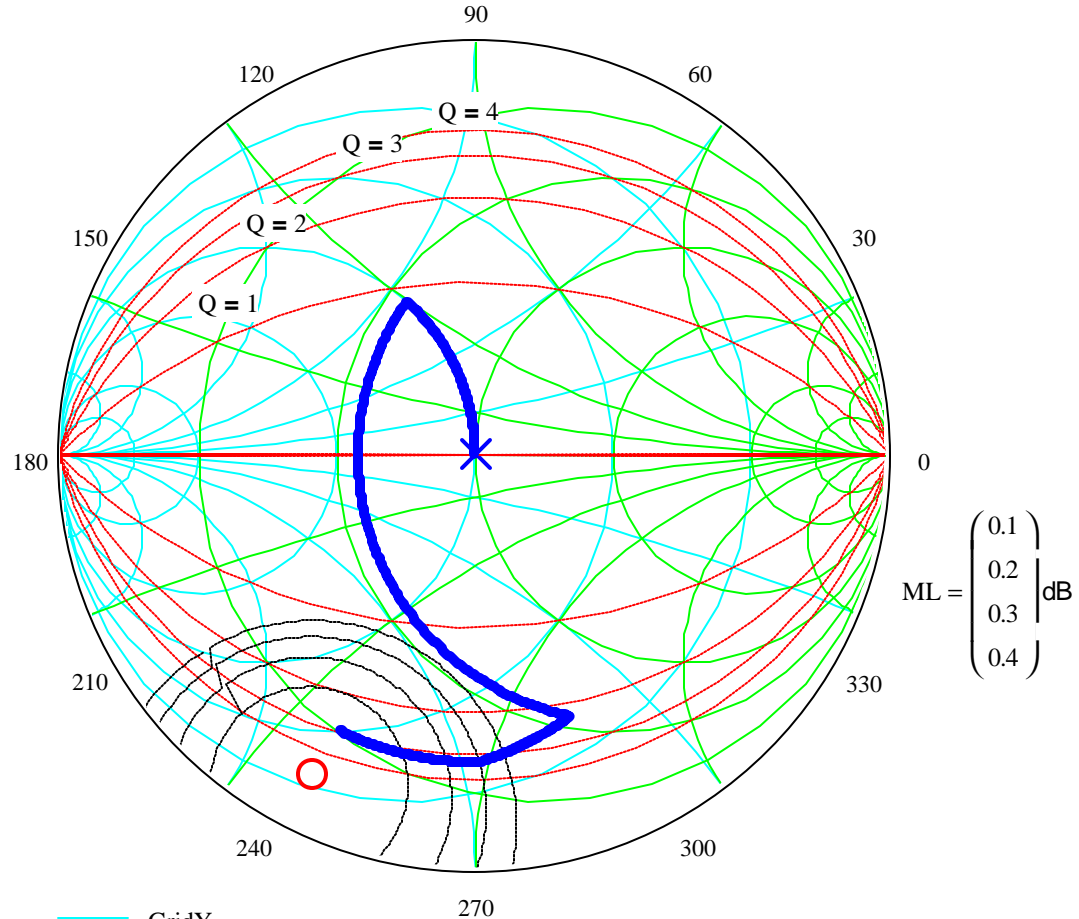
Curve Generation

Grid

m =	$\begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{pmatrix}$
sp =	$\begin{pmatrix} "p" \\ "s" \\ "p" \\ "s" \\ "p" \\ "p" \\ "p" \\ "s" \end{pmatrix}$
LCtype =	$\begin{pmatrix} "L" \\ "C" \\ "C" \\ "R" \\ "R" \\ "T" \\ "T" \\ "T" \end{pmatrix}$
LC =	$\begin{pmatrix} 1 \times 10^{-8} \\ 2 \times 10^{-12} \\ 1 \times 10^{-12} \\ 0.01 \\ 1 \times 10^6 \\ 50 \\ 50 \\ 50 \end{pmatrix}$
$\gamma =$	$\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1.572i \\ 1.131i \times 10^{-4} \\ 0.226i \end{pmatrix}$
$Z_{\text{end}} =$	$\begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \cdot 10^6 \\ 0 \end{pmatrix} \Omega$

(9) ("s") ("L") $\left[1 \times 10^{-10} \right]$ (0.2201)

9	0
10	0



$$ML = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \end{pmatrix} \text{ dB}$$

- GridY
- GridZ
- X X Source Impedance
- O O Load Conjugate
- Constant Q Contour
- Constant VSWR Contours
- Matching Elements

S Parameter Frequency Response

S-parameter is an abbreviation for scattering parameters. S-parameters are a measure of the power gain of a network. The term scattering comes from the concept of a cue ball scattering other balls as it transfers power to them. S_{ij} is the measure of power gain from port j to port i . Specifically, the square root of power gain. For example, S_{21} , the most useful S-parameter, is the measure of the ratio of output power to the available input power. This is an important sentence. This sentence is used to perform hand calculations. The output power is easy to explain, it is V_{Orms}^2/R_L . The available input power is trickier to explain. Available input power is the maximum power that can be delivered from a source. For a source impedance of R_S , and a source voltage of V_{Srms} ; the available input power is $(V_{Srms}/2)^2/R_S$. Why, because maximum power is delivered is to a resistance of R_S . Thus a voltage division of two for the voltage, when delivering maximum available power. Thus the power gain of a network, S_{21} , is found by dividing the two powers $2 \cdot V_o/V_s \cdot \sqrt{R_S/R_L}$.

S-parameters are most conveniently calculated by converting an impedance or admittance matrix into a S-parameter matrix. So we start by finding the admittance (Y) parameters for the lowpass L matching network. Admittance (Y) parameters are found by driving a port with a voltage source, shorting the rest, and measuring the current at all the ports.

$$\begin{aligned}
 \text{ABCD}(\omega) := & \left| \begin{array}{l}
 \text{ABCD}_{\text{prev}} \leftarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\
 \text{for } m \in 1..(\text{length}(\text{LC}) - 1) \\
 \quad \left| \begin{array}{l}
 \text{Zx} \leftarrow \frac{1}{j \cdot \omega \cdot \text{LC}_m \cdot F \cdot \Omega} \text{ if } \text{LCtype}_m = \text{"C"} \\
 \text{Zx} \leftarrow j \cdot \omega \cdot \text{LC}_m \cdot \frac{H}{\Omega} \text{ if } \text{LCtype}_m = \text{"L"} \\
 \text{Zx} \leftarrow \text{LC}_m \text{ if } \text{LCtype}_m = \text{"R"} \\
 \text{Zx} \leftarrow \text{LC}_m \cdot \frac{\text{Z}_{\text{end}_m} + \text{LC}_m \cdot \Omega \cdot \tanh(\gamma_m)}{\text{LC}_m \cdot \Omega + \text{Z}_{\text{end}_m} \cdot \tanh(\gamma_m)} \text{ if } \text{LCtype}_m = \text{"T"} \\
 \text{ABCDx} \leftarrow \begin{pmatrix} 1 & \text{Zx} \\ 0 & 1 \end{pmatrix} \text{ if } (\text{sp}_m = \text{"s"}) \cdot (\text{LCtype} \neq \text{"T"}) \\
 \text{ABCDx} \leftarrow \begin{pmatrix} 1 & 0 \\ \frac{1}{\text{Zx}} & 1 \end{pmatrix} \text{ if } (\text{sp}_m = \text{"p"}) \\
 \text{ABCDx} \leftarrow \begin{pmatrix} \cosh(\gamma_m) & \text{LC}_m \cdot \sinh(\gamma_m) \\ \frac{1}{\text{LC}_m} \cdot \sinh(\gamma_m) & \cosh(\gamma_m) \end{pmatrix} \text{ if } (\text{sp}_m = \text{"s"}) \cdot (\text{LCtype}_m = \text{"T"}) \\
 \text{ABCD} \leftarrow \text{ABCDx} \cdot \text{ABCD}_{\text{prev}} \\
 \text{ABCD}_{\text{prev}} \leftarrow \text{ABCD}
 \end{array} \right. \\
 \text{ABCD}
 \end{array}
 \right.
 \end{aligned}$$

ABCD to S parameter conversion with a common impedance on all ports using this expression from "Microwave Engineering" by Pozar.

$$\text{ABCD2S}(\text{ABCD}, Z_0) := \left| \begin{array}{l}
 A \leftarrow \text{ABCD}_{1,1} \\
 B \leftarrow \text{ABCD}_{1,2}
 \end{array} \right.$$

$$\begin{aligned}
 & C \leftarrow ABCD_{2,1} \\
 & D \leftarrow ABCD_{2,2} \\
 & \left[\begin{array}{cc}
 \frac{1}{\left(A + \frac{B \cdot \Omega}{Z_0} + C \cdot \frac{Z_0}{\Omega} + D \right)} \cdot \left[\begin{array}{cc}
 A + \frac{B \cdot \Omega}{Z_0} - C \cdot \frac{Z_0}{\Omega} - D & 2 \cdot (A \cdot D - B \cdot C) \\
 -A + \frac{B \cdot \Omega}{Z_0} - C \cdot \frac{Z_0}{\Omega} + D & 2
 \end{array} \right] & \text{if } \left(A + \frac{B \cdot \Omega}{Z_0} + C \cdot \frac{Z_0}{\Omega} + D \right) \neq 0 \\
 \begin{pmatrix} 1 & 10^8 \\ 0 & 1 \end{pmatrix} & \text{if } \left(A + \frac{B \cdot \Omega}{Z_0} + C \cdot \frac{Z_0}{\Omega} + D \right) = 0
 \end{array} \right]
 \end{aligned}$$

This scattering parameter conversion routine (to convert to arbitrary source and load impedances) is given "Applied RF Techniques I" lecture notes, but is incorrect. A correct version of the routine is found on page 31 of "Microwave Amplifiers and Oscillators," by Christian Gentili.

$$S_{\text{conv}}(S, Z_{\text{Sbegin}}, Z_{\text{Send}}, Z_{\text{Lbegin}}, Z_{\text{Lend}}) := \left[\begin{array}{cc}
 \Gamma_S \leftarrow \frac{Z_{\text{Send}} - Z_{\text{Sbegin}}}{Z_{\text{Send}} + Z_{\text{Sbegin}}} & \\
 \Gamma_L \leftarrow \frac{Z_{\text{Lend}} - Z_{\text{Lbegin}}}{Z_{\text{Lend}} + Z_{\text{Lbegin}}} & \\
 D \leftarrow (1 - \Gamma_S \cdot S_{1,1}) \cdot (1 - \Gamma_L \cdot S_{2,2}) - \Gamma_S \cdot \Gamma_L \cdot S_{1,2} \cdot S_{2,1} & \\
 A_1 \leftarrow \frac{1 - \overline{\Gamma_S}}{|1 - \Gamma_S|} \cdot \sqrt{1 - (|\Gamma_S|)^2} & \\
 A_2 \leftarrow \frac{1 - \overline{\Gamma_L}}{|1 - \Gamma_L|} \cdot \sqrt{1 - (|\Gamma_L|)^2} & \\
 \left[\begin{array}{cc}
 \frac{\overline{A_1}}{A_1} \cdot \frac{(1 - \Gamma_L \cdot S_{2,2}) \cdot (S_{1,1} - \overline{\Gamma_S}) + \Gamma_L \cdot S_{1,2} \cdot S_{2,1}}{D} & \frac{\overline{A_2}}{A_1} \cdot \frac{S_{1,2} \cdot [1 - (|\Gamma_S|)]}{D} \\
 \frac{\overline{A_1}}{A_2} \cdot S_{2,1} \cdot \frac{1 - (|\Gamma_L|)^2}{D} & \frac{\overline{A_2}}{A_2} \cdot \frac{(1 - \Gamma_S \cdot S_{1,1}) \cdot (S_{2,2} - \overline{\Gamma_L})}{D}
 \end{array} \right]
 \end{array} \right.$$

$$S_{50}(\omega) := ABCD2S(ABCD(\omega), Z_0)$$

S-parameters of ideal matching network with actual load and source impedances.

$$S(\omega) := S_{\text{conv}}(S_{50}(\omega), Z_0, Z_S, Z_0, Z_L)$$

50 Ohm S-parameters of ideal matching network.

Plots of lossy and ideal S-paramaters of the matching network verses frequency. Be careful when using these plots, as they do not reflect the change in source and load impedance vs. frequency (for example if driver output impedance is capacitive).

$$f_{\text{start}} := \frac{f}{5}$$

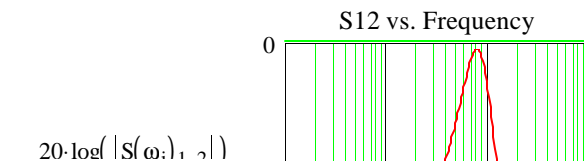
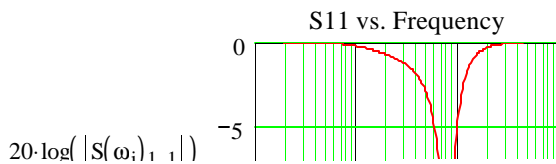
$$f_{\text{stop}} := f \cdot 50$$

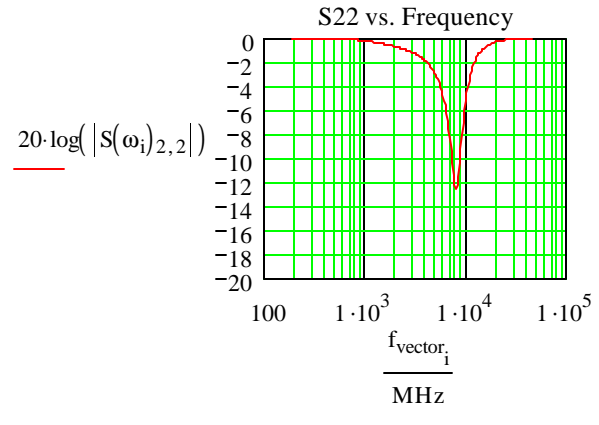
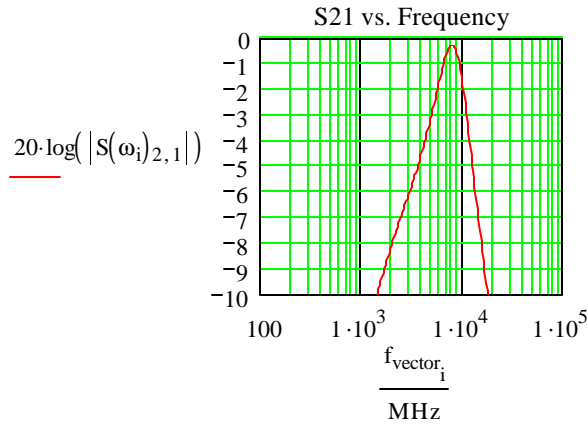
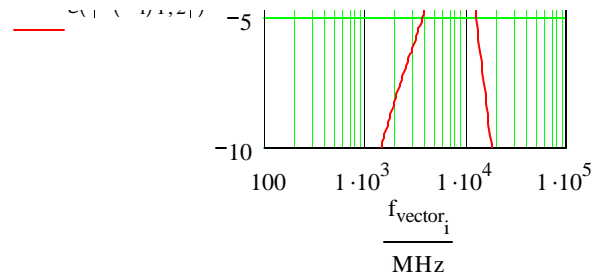
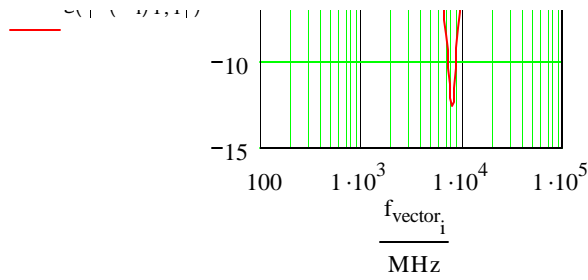
Starting and stopping frequencies for plot

$$f_{\text{vector}_i} := f_{\text{start}} \cdot \left(\frac{f_{\text{stop}}}{f_{\text{start}}} \right)^{\frac{i}{\text{numi}}}$$

$$\omega_i := 2 \cdot \pi \cdot f_{\text{vector}_i}$$

$$s_i := j \cdot \omega_i$$





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