Mirowave Transistors

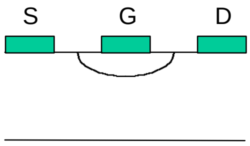
Frequency (GHz)	GaAs FET		GaAs HEMT		Silicon Bipolar		GaAs HBT	
	Gain	F_{min}	Gain	F_{min}	Gain	F_{min}	Gain	F_{min}
4	20	0.5	—	—	15	2.5	—	—
8	16	0.7	—	—	9	4.5	—	—
12	12	1.0	22	0.5	6	8.0	20	4.0
18	8	1.2	16	0.9	—	—	16	—
36	—	—	12	1.7	—	—	10	—
60	—	—	8	2.6	—	—	7	—

Modeling method

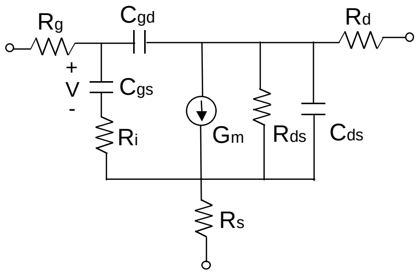
- S-parameters : up to meas. freqs.
- Equivalent ckt
  - freq. extrapolation
  - physical

GaAs FET

Physical structure



Intrinsic EC model(no pad)

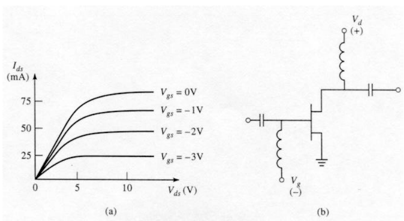


f_T(current gain cut-off freq)

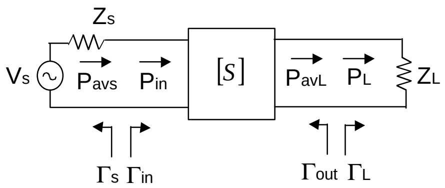
$$H_{21} = \left|\frac{I_d}{I_g}\right| \approx \left|\frac{g_m V}{wC_{gs} V}\right| \approx \frac{g_m}{wC_{gs}}$$

$$f_T = \frac{g_m}{2pC_{gs}}$$

Biasing scheme



Power Gains



Power Gain Definition

- Power gain = G

  $G = \frac{P_L}{P_{in}}$: indep of Zs

- Available gain = GA

  $G_A = \frac{P_{avL}}{P_{avs}}$: indep of ZL

- Transducer Power gain = GT

  $G_T = \frac{P_L}{P_{avs}}$: dep on Zs and ZL

Formula

$$G = \frac{|S_{21}|^2 (1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)|1-S_{22}\Gamma_L|^2}$$

$$G_A = \frac{|S_{21}|^2 (1-|\Gamma_S|^2)}{|1-S_{11}\Gamma_S|^2 (1-|\Gamma_{out}|^2)}$$

$$G_T = \frac{|S_{21}|^2 (1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-\Gamma_S\Gamma_{in}|^2 |1-S_{22}\Gamma_L|^2}$$

$$G_{TU} = G_T\big|_{S_{12}=0} = \frac{|S_{21}|^2 (1-|\Gamma_S|^2)(1-|\Gamma_L|^2)}{|1-S_{11}\Gamma_S|^2 |1-S_{22}\Gamma_L|^2}$$

Amplifier Gain - S-parameter Approach

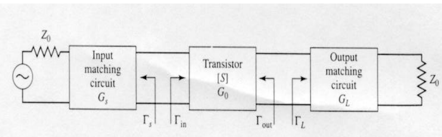


FIGURE 11.8 The general transistor amplifier circuit.

- $G_{Ttot} = \frac{P_{L,S}}{P_{avs}} \cdot \frac{P_{out,D}}{P_{in,D}} \cdot \frac{P_{L,L}}{P_{in,L}}$

$$= [G_T]_S \cdot [G]_D \cdot [G]_L$$

$$[G_T]_S = \frac{|S_{21S}|^2 (1-|\Gamma_{in}|^2)}{|1-S_{22S}\Gamma_{in}|^2}$$

$$[G]_D = \frac{|S_{21}|^2 (1-|\Gamma_L|^2)}{(1-|\Gamma_{in}|^2)|1-S_{22}\Gamma_L|^2}$$

$$[G]_L = \frac{|S_{21L}|^2}{1-|\Gamma_L|^2}$$

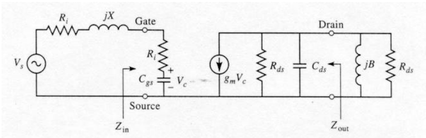
$$G_{Ttot} = \frac{|S_{21S}|^2 |S_{21}|^2 |S_{21L}|^2}{|1-S_{22S}\Gamma_{in}|^2 |1-S_{22}\Gamma_L|^2}$$

$$S_{22S} = \Gamma_S \rightarrow |S_{21S}|^2 = 1-|\Gamma_S|^2$$

$$S_{11L} = \Gamma_L \rightarrow |S_{21L}|^2 = 1-|\Gamma_L|^2$$

$$G_{Ttot} = \frac{(1-|\Gamma_S|^2)|S_{21}|^2 (1-|\Gamma_L|^2)}{|1-\Gamma_{in}\Gamma_S|^2 |1-S_{22}\Gamma_L|^2}$$

Amplifier Gain - from Equivalent Circuit



Matching condition (Assume C_{gd}=0 $\longrightarrow$ S_{12}=0)

$$Z_{in} = Z_S^* \Rightarrow X = \frac{1}{wC_{gs}}$$

$$Z_{out} = Z_L^* \Rightarrow B = -wC_{ds}$$

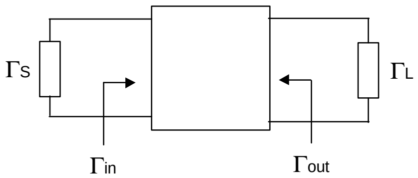
- $$G_{TU} = \frac{\frac{1}{8}|g_m V_c|^2 R_{ds}}{\frac{1}{8}|V_s|^2 / R_i} = \frac{g_m^2 R_{ds}}{4w^2 R_i C_{gs}^2} = \frac{R_{ds}}{4R_i}\left(\frac{f_T}{f}\right)^2$$

$\Rightarrow 6dB/oct\ roll-off$

Maximum frequency of oscillation

$f_{\max} = \frac{f_T}{2}\sqrt{\frac{R_{ds}}{R_{in}}} \rightarrow$ most important parameter for Amp., Osc.

Stability



$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}}{1 - S_{22}\Gamma_L}$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1 - S_{11}\Gamma_S}$$

∴ for some ΓS, ΓL, Γin, Γout can be greater than 1

⇨ unstable!

Stability Circle

$|\Gamma_{in}| = 1$ circle in ΓL - plane

⇨ output stability circle

$|\Gamma_{out}| = 1$ circle in ΓS - plane

⇨ input stability circle

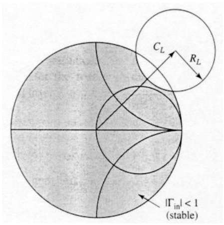
for $|\Gamma_{in}| = 1$

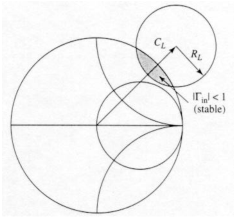
$$\left|\Gamma_L - \frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2}\right| = \left|\frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2}\right|$$

⇨ Radius(RL) = $\left|\frac{S_{12}S_{21}}{|S_{22}|^2 - |\Delta|^2}\right|$

Center(CL) = $\frac{(S_{22} - \Delta S_{11}^*)^*}{|S_{22}|^2 - |\Delta|^2}$

Stability Circle





Stable region?(output)

i) $|S_{11}| < 1$

⇨ for $\Gamma_L = 0$, $\Gamma_{in} = S_{11} < 1$

⇨ stable for region containing $\Gamma_L = 0$

ii) $|S_{11}| > 1$

⇨ stable for region excluding $\Gamma_L = 0$

Amplifier Stability

Stability

(1) Unconditional Stability

$$\Rightarrow for\ \ all\ \ \Gamma_s\ \ and\ \ \Gamma_l$$
$$|\Gamma_{in}|\ ,\ |\Gamma_{out}| < 1$$

(2) Conditional Stability

$$\Rightarrow for\ \ some\ \ \Gamma_s\ \ and\ \ \Gamma_l$$
$$|\Gamma_{in}|\ ,\ |\Gamma_{out}| < 1$$

Conditions (iff)

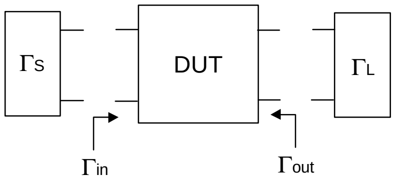
$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|^2} > 1$$

$$where,\ \ \Delta = S_{11}S_{22} - S_{12}S_{21}$$

Conditional Stable

$$\Rightarrow Need\ to\ choose\ \ \Gamma_s\ ,\ \Gamma_l$$
$$for\ stable\ region$$

Narrowband Amplifier Design



$$G_T = \frac{(1-|\Gamma_S|^2)}{|1-\Gamma_S\Gamma_{in}|^2}\cdot|S_{21}|^2\cdot\frac{(1-|\Gamma_L|^2)}{|1-S_{22}\Gamma_L|^2}$$

$$= G_S \cdot G_0 \cdot G_L$$

$$G_S = \frac{1-|\Gamma_S|^2}{|1-\Gamma_S\Gamma_{in}|^2}$$

$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

Maximum gain

$$\begin{cases}\Gamma_{in} = \Gamma_S^* \\ \Gamma_{out} = \Gamma_L^*\end{cases}$$

$$G_{T\max} = \frac{1}{1-|\Gamma_S|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

Condition for simultaneous match

$$\Gamma_S^* = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L}$$

$$\Gamma_L^* = S_{22} + \frac{S_{12}S_{21}\Gamma_S}{1-S_{11}\Gamma_S}$$

$$\Rightarrow \begin{cases}\Gamma_S = \dfrac{B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1} \\ \Gamma_L = \dfrac{B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}\end{cases}$$

where $B_1 = 1+|S_{11}|^2 - |S_{22}|^2 - |\Delta|^2 \quad C_1 = S_{11} - \Delta S_{22}^*$

$B_2 = 1+|S_{22}|^2 - |S_{11}|^2 - |\Delta|^2 \quad C_2 = S_{22} - \Delta S_{11}^*$

Unilateral Case

Unilateral Transducer Gain

$$\Gamma_{in} = S_{11} \quad , \quad \Gamma_{out} = S_{22}$$

$$G_{TU} = \frac{|S_{21}|^2(1-|\Gamma_s|^2)\ (1-|\Gamma_l|^2)}{|1-S_{11}\Gamma_s|^2\ |1-S_{22}\Gamma_l|^2}$$

Condition for Simultaneous Match

$$\Gamma_s^* = S_{11} \quad , \quad \Gamma_l^* = S_{22}$$

Maximum Unilateral Gain

$$G_{TU_{\max}} = \frac{1}{1-|S_{11}|^2}\ |S_{21}|^2\ \frac{1}{1-|S_{22}|^2}$$

Mismatched Amp Design - Gain Circles(unilateral)

Gain for unilateral case

$$G = G_S \cdot |S_{21}|^2 \cdot G_L$$

$$G_S = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}$$

$$G_L = \frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

Maximum unilateral gain

$$G_{S\max} = G_S\big|_{\Gamma_S = S_{11}^*} = \frac{1}{1-|S_{11}|^2}$$

$$G_{L\max} = G_L\big|_{\Gamma_L = S_{22}^*} = \frac{1}{1-|S_{22}|^2}$$

Gain factor g_S, g_L

$$G = G_{S\max} \cdot |S_{21}|^2 \cdot G_{L\max} \cdot \frac{G_S}{G_{S\max}} \cdot \frac{G_L}{G_{L\max}}$$

$$= G_{\max} \cdot g_S \cdot g_L$$

where $g_S = \dfrac{G_S}{G_{S\max}}$, $g_L = \dfrac{G_L}{G_{L\max}}$

Gain circles

For given g_S, find Γ_S

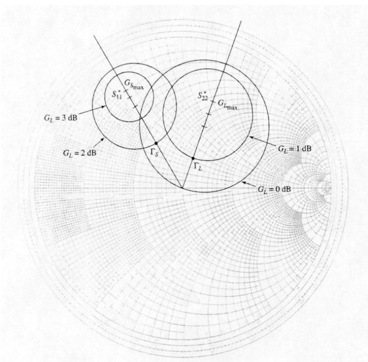
$$g_S = \frac{G_S}{G_{S\max}} = \frac{1-|\Gamma_S|^2}{|1-S_{11}\Gamma_S|^2}(1-|S_{11}|^2)$$

⇨ $$\begin{cases} C_S = \dfrac{g_S S_{11}^*}{1-(1-g_S)|S_{11}|^2} \\ R_S = \dfrac{\sqrt{1-g_S}\,(1-|S_{11}|^2)}{1-(1-g_S)|S_{11}|^2} \end{cases}$$

⇨ Circles!

- center : on the line to S^*_{11}
- $\Gamma_S=0 \rightarrow G_S=1$ ⇨ 0dB circle cross the center

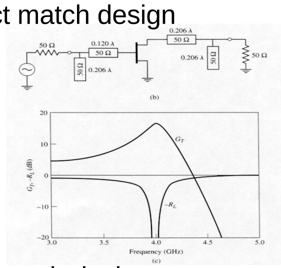
Gain Circle and Bandwidth



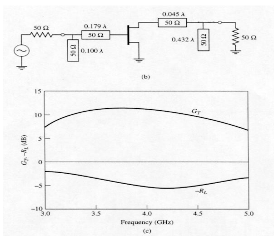
⇨ Choose Γ_S and Γ_L close to the center of Smith chart

⇨ for max bandwidth

1. Exact match design



2. Mismatch design



Low Noise Amplifier Design

Noise figure of 2-port device

$$F = F_{\min} + \frac{R_N}{G_S}\left|Y_S - Y_{opt}\right|^2$$

where $Y_S = G_S + jB_S, Y_{opt} = G_{opt} + jB_{opt}$

4 noise parameters

¨ çF_{min} ¨ èR_n

¨ éG_{opt} ¨ êB_{opt}

S-parameter representation

$$F = F_{\min} + \frac{4R_N}{Z_0}\frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{(1-\left|\Gamma_S\right|^2)\left|1+\Gamma_{opt}\right|^2}$$

Noise figure parameters

⇨ circles

$$N = \frac{F - F_{\min}}{4R_N/Z_0}\left|1+\Gamma_{opt}\right|^2$$

⇨ determined by noise parameters

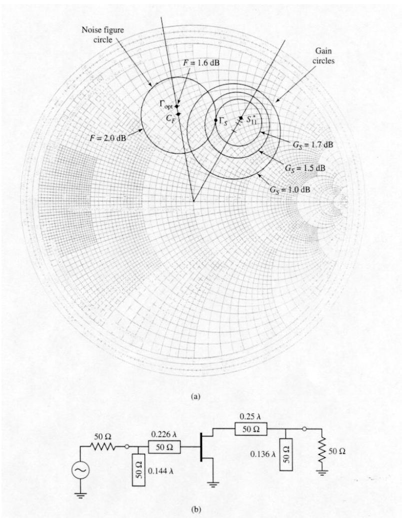
Constant Noise figure circles

$$N = \frac{\left|\Gamma_S - \Gamma_{opt}\right|^2}{1-\left|\Gamma_S\right|^2}$$

⇨ $$\left|\Gamma_S - \frac{\Gamma_{opt}}{N+1}\right| = \frac{\sqrt{N(N+1-\left|\Gamma_{opt}\right|^2)}}{(N+1)}$$

∗ Circle center on a straight line from the center to Γ_{opt}

Low Noise Amplifier Design(cont d)



Amplifier design

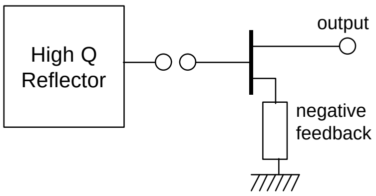
RL, Stability, Bandwidth

LNA design

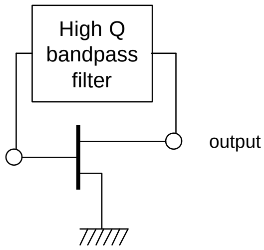
RL, Stability, NF, Bandwidth

Compromise between NF, RL, Stability and Gain

Oscillator - Types



<Reflection type>



<Transmission type>

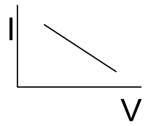
Two types of VCO

Transmission type : amplifier mode ⇨ straightforward

Reflection type : negative resistance mode : less feed structure
⇨ potentially higher Q factor

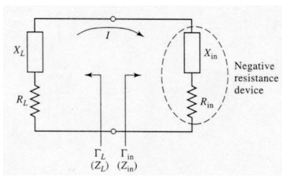
One - port Negative Resistance Oscillator

Negative Resistance Device



Gunn diode , IMPATT, TUNNET

Circuit



KVL

$\rightarrow (Z_L + Z_{in})I = 0$

$\Rightarrow \; R_L + R_{in} = 0$

$\quad X_L + X_{in} = 0$

S-parameter representation

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{-Z_{in} - Z_0}{-Z_{in} + Z_0} = \frac{1}{\Gamma_L}$$

$$\Rightarrow \Gamma_L \cdot \Gamma_{in} = 1 \quad at \quad s.s$$

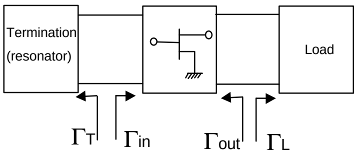
Stable oscillation with perturbations in I or frequency, go back to original oscillation status

$$Z_{in}(I,w), Z_L(w)$$

$$\Rightarrow \frac{\partial (X_L + X_{in})}{\partial w} >> 0$$

$$\therefore High\,Q\ resonator\ needed$$

Transistor Oscillator



Design procedure

- Choose Γ_T for max Γ_{out} (>1)

  $Z_{out} = R_{out} + jX_{out}$

  $\Rightarrow Maximize \left|R_{out}\right| (negative)$

- Choose $Z_L = R_L + jX_L$

  $R_L = -\frac{R_{out}}{3} \; (for \; max \; power)$

  $X_L = -X_{out}$

Simultaneous Oscillation
- output oscilltor

$$\Gamma_L \cdot \Gamma_{out} = 1$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_T}{1 - S_{11}\Gamma_T} = \frac{S_{22} - \Delta\Gamma_T}{1 - S_{11}\Gamma_T}$$

$$\Gamma_T = \frac{1 - S_{22}\Gamma_L}{S_{11} - \Delta\Gamma_L}$$

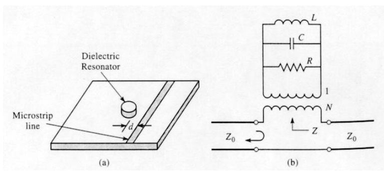
$$\Gamma_{in} = \frac{S_{11} - \Delta\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\therefore \Gamma_T \cdot \Gamma_{in} = 1$$

⇨ Input oscillation

(Large signal S-parameter)

DRO:Dielectric Resonator Oscillator



$$Z = \frac{N^2R}{1 + j2Q\Delta w / w_0} (equivalent \; series \; impedance)$$

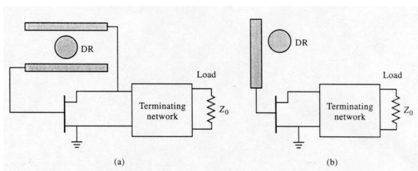
- *Coupling factor*

$$g = \frac{Q}{Q_e} = \frac{R / w_0 L}{R_L / N^2 w_0 L} (R_L = 2Z_0)$$

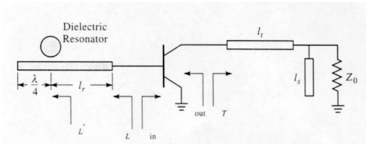
- Γ *at the resonator plane*

$$\Gamma = \frac{(Z_0 + N^2R) - Z_0}{Z_0 + N^2R + Z_0} = \frac{g}{1+g}$$

$$\therefore Measure \Gamma \rightarrow find \; g \rightarrow find \; N^2R$$



DRO Design



Design Procedure

- Find Γ_L for max $|R_{out}|$
- Design output matching circuit

$$Z_T = -\frac{R_{out}}{3} - jX_{out}$$

- Transform Γ_L to Γ_L'

$$\Gamma_L' = \Gamma_L e^{j2bl_r}$$

- Find Z_L'

$$Z_L' = Z_0 \frac{1+\Gamma_L'}{1-\Gamma_L'}$$

- Find g

$$g = \frac{Z_L'}{Z_0}$$ (note : $Z_L = Z_0$)

Γ_{out} vs frequency (effect of DR)