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**Inputs**

- \( Z_S := 200 \text{ohm} - j50 \text{ohm} \)  
  Source impedance

- \( Z_L := 50 \text{ohm} + j25 \text{ohm} \)  
  Load impedance

- \( f_{\min} := 900 \text{MHz} \)  
  Lower edge of band frequency

- \( f_{\max} := 900 \text{MHz} \)  
  Upper edge of band frequency
Solving this expression gives a general expression for the $Q$ of matching network. Note the $Q$ value is fixed by the source and load impedance for the $L$ matching network. For higher order networks, and for equal source and load resistance, the $Q$ of the network may be chosen.

$$R_p + j \frac{Q S}{\omega} = R_S + j X_{right} = R_S \left(1 - j Q_{right}\right)$$

Because the left and right resistances must be equal, and the reactances conjugates of each other, the $Q$’s of the left and right sides must be equal. Setting the left impedance and right conjugate equal to each other, and setting the $Q$’s equal gives the following equality.

$$\frac{R_p}{1 + j Q} = R_S (1 - j Q)$$

Solving this expression gives a general expression for the $Q$ of matching network. Note the $Q$ value is fixed by the source and load impedance for the $L$ matching network. For higher order networks, and for equal source and load resistance, the $Q$ of the network may be chosen.

Calculations

The $L$ matching network is only designed to provide maximum power transfer at one frequency. In other words it is a narrow band matching network. Given a band of input frequencies, we set design the matching network for the center of the band.

$$f_c := \frac{f_{\min} + f_{\max}}{2} \quad f_c = 900 \text{MHz}$$

The first step in synthesizing an impedance matching network is to deimbed the source and load reactances. Because the matching network consists of a capacitor in series with the load, the load impedance is converted into an equivalent series resistance and capacitance. The equivalent series capacitance and resistance of load impedance are:

$$C_1 := \begin{cases} \text{Im}(Z_L) = 0 \text{ohm}, & 1000 \text{F}, \quad -1 \frac{1}{\omega} \text{Im}(Z_L) \end{cases} \quad C_1 = -7.074 \text{pF}$$

$$R_S := \text{Re}(Z_L) \quad R_S = 50 \text{ohm}$$

Deimbedding the source reactance is a little more difficult, because the impedance is parallel. The calculation is easy to perform by converting the input impedance to an admittance with a complex math processor, such as Mathcad, but is performed here assuming only a real processor is available. The equivalent parallel inductance and resistance of source impedance are:

$$Q_S := \frac{\text{Im}(Z_S)}{\text{Re}(Z_S)} \quad Q_S = -0.25$$

$$R_p := \left(1 + Q_S^2\right) \frac{\text{Re}(Z_S)}{\text{Im}(Z_S)} \quad R_p = 212.5 \text{ohm}$$

$$L_1 := \left[Q_S = 0, 1000 \text{H}, \quad 1 + Q_S^2 \frac{\text{Re}(Z_S)}{\text{Im}(Z_S)} \right] \quad L_1 = -150.313 \text{nH}$$

With the source and load reactances deimbedded, we can synthesize the matching network using the real part of the source and the load. Our goal here is to size the $L$ and the $C$ such that source and the load sees the conjugate of it's impedance looking into the network. A matched network sliced anywhere in the network will have a conjugate impedance looking in either direction.

$$Z_{left} = Z_{right}$$

The left side of the matching network consists of a parallel resistance and reactance. This can be expressed in a simplified form with the parallel resistance and the $Q$ of the source.

$$Z_{left} = \frac{1}{R_{left} + \frac{1}{1 + \frac{1}{j Q_{left}}} \left(1 + Q_{left} \right)}$$

The right side of the matching network consists of a series resistance and reactance. This can be expressed in a simplified form with the series resistance and the $Q$ of the load.

$$Z_{right} = R_{right} + \frac{1}{j C_{right}} = R_S + j X_{right} = R_S \left(1 - j Q_{right}\right)$$

Because the left and right resistances must be equal, and the reactances conjugates of each other, the $Q$'s of the left and right sides must be equal. Setting the left impedance and right conjugate equal to each other, and setting the $Q$'s equal gives the following equality.

$$\frac{R_p}{1 + j Q} = R_S (1 - j Q)$$
\[ Q := \frac{\sqrt{R_S}}{R_p} - 1 \quad Q = 1.803 \]

This Q will serve as a lower bound for \( q \) and T matching networks, and as an upper bound for cascaded L matching networks. Note the difference between network Q and component Q. Network Q is usually on the order of zero to three, to reduce sensitivity of the network. Component Q is much higher, typically 10 to 100, to

Given the Q of the left and right half sides of the network, and the effective source and load impedances, we can find the matching network inductance and capacitance without source inductance deembedded

\[ C_S := \frac{1}{Q \omega R_S} \quad C_S = 1.962 \ \text{pF} \quad L_p := \frac{R_p}{\omega Q} \quad L_p = 20.845 \ \text{nH} \]

The final impedance network is found by subtracting the source inductance from the matching inductance and the load capacitance from the matching capacitance.

\[ C := \begin{cases} C_1 = C_S, & 1000 \text{F}, \frac{C_1 \cdot C_S}{C_1 - C_S} \quad C = 1.536 \ \text{pF} \\ L := \begin{cases} L_1 = L_p, & 1000 \text{H}, \frac{L_p \cdot L_1}{L_1 - L_p} \quad L = 18.306 \ \text{nH} \end{cases} \]

We can check our calculation, by checking to see if the input and output impedances are complex conjugates of each other.

\[ Z_i := (j \omega L) \parallel \left( Z_L + \frac{1}{j \omega C} \right) \quad Z_i = 200 + 50 \ \text{ohm} \quad Z_S = 200 - 50 \ \text{i} \ \Omega \]

\[ Z_o := Z_S \parallel (j \omega L) + \frac{1}{j \omega C} \quad Z_o = 50 - 25 \ \text{i} \ \text{ohm} \quad Z_L = 50 + 25 \ \text{i} \ \text{\Omega} \]

### Outputs

- \( C = 1.536 \ \text{pF} \)
- \( L = 18.306 \ \text{nH} \)
- \( \text{Load Capacitance (in series with load)} \)
- \( \text{Source Inductance (in parallel with source)} \)

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