Highpass L Matching Network Designer
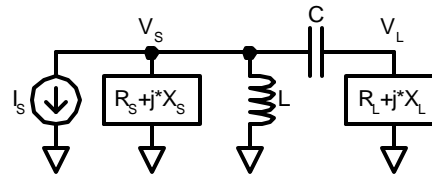

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General Impedance Matching

This worksheets discusses in detail the impedance matching with application to the synthesis of a highpass L-shaped impedance matching network. The design of the matching networks is performed in many textbooks using a smith chart. In this report, the design is performed

$$P_S = \frac{V_{Srms}^2}{R_S} \qquad P_L = \frac{V_{Lrms}^2}{R_L} \qquad V_{Lrms} = V_{Srms} \cdot \frac{Z_L}{Z_S + Z_L}$$

The power gain of the network is a found by dividing the expression for the output power divided by the input power.

$$G_p = \frac{P_L}{P_S} = \left(\frac{Z_L}{Z_S + Z_L} \right)^2 \cdot \frac{R_S}{R_L} = \left[\frac{R_L^2 + X_L^2}{(R_S + R_L)^2 + (X_L + X_S)^2} \right] \cdot \frac{R_S}{R_L}$$

To optimize the power gain for a given load impedance the derivative of power gain with respect to the source resistance and reactance is set to zero, and the source impedance is solved. This is easiest done in a two step process by first finding the optimal source reactance, $\frac{d}{dX_S} G_p = 0$, which gives an optimal source reactance of:

$$X_{Sopt} = -X_L$$

The source resistance can be found similarly with $\frac{d}{dR_S} G_p = 0$, giving an optimal source resistance of:

$$R_{Sopt} = R_L$$

This impedance is also said to give zero power reflection. That is all of the power is transferred to the load, and none is reflected back to source. When viewing the power signals as waves it is difficult to view a reflection. when

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the matching network consists of discrete elements. When studying distributed structures, such as transmission lines, we see the find that zero reflection occurs for true matched line and load impedance (not conjugate). So if you are delivering power from a source through an input matching network, through a transmission line, through an output matching network, and finally to a load, how is that you can provide maximum power delivery to the load with a conjugate match, but have zero reflections? The answer is: "You can't, " because this situation does not occur in practice. In practice in order for a transmission line to have an imaginary characteristic impedance it must be lossy.

Impedance Transformation for Power Amplifiers

Careful! Output networks for power amplifiers are more commonly "impedance transformation" networks rather than "impedance matching" networks. The goal of a power amplifier is to efficiently transfer power from a **DC** power supply to a load at an RF frequency; however an impedance matching network provides maximum power transfer from an **RF** source to a load. In turns out the two are the same for output powers less than $(V_{DD}-V_{DSsat})^2/2/R_L$, which is around 100mW for a 50 ohm load and 3V supply. For powers above this, the load is **transformed** to an impedance, seen from the power amplifier, which is much lower than the output impedance of the PA.

▶ useful functions and identities
▶ Units
▶ Constants

Inputs

$Z_S := 200\text{ohm} - j \cdot 50\text{ohm}$ Source impedance

$Z_L := 50\text{ohm} + j \cdot 25\text{ohm}$ Load impedance

$f_{min} := 900\text{MHz}$ Lower edge of band frequency

$f_{max} := 900\text{MHz}$ Upper edge of band frequency

$I_i := 100\text{mA}_{rms}$ Input current

$V_{imax} := 4V_{rms}$ Input voltage constraint $V_i < V_{imax}$

$V_{omax} := 4V_{rms}$ Output voltage constraint $V_o < V_{imax}$

Calculations

The L matching network is only designed to provide maximum power transfer at one frequency. In other words it is a narrow band matching network. Given a band of input frequencies, we set design the matching network for the center of the band.

$$f_c := \frac{f_{min} + f_{max}}{2} \qquad f_c = 900\,\text{MHz}$$

$$f := f_c \qquad \omega_c := 2 \cdot \pi \cdot f_c \qquad \omega := 2 \cdot \pi \cdot f$$

The first step in synthesizing an impedance matching network is to deimbedd the source and load reactances. Because the matching network consists of a capacitor in series with the load, the load impedance is converted into an equivalent series resistance and capacitance. The equivalent series capacitance and resistance of load impedance are:

$$C_1 := \text{if}\left(\text{Im}(Z_L) = 0\,\text{ohm}, 1000\text{F}, \frac{-1}{\omega \cdot \text{Im}(Z_L)} \right) \qquad C_1 = -7.074\,\text{pF}$$

$$R_S := \text{Re}(Z_L) \qquad\qquad R_S = 50\,\Omega$$

Deimbedding the source reactance is a little more difficult, because the impedance is parallel. The calculation is easy to perform by converting the input impedance to an admittance with a complex math processor, such as Mathcad, but is performed here assuming only a real processor is available. The equivalent parallel inductance and resistance of source impedance are:

$$Q_S := \frac{\text{Im}(Z_S)}{\text{Re}(Z_S)} \qquad\qquad Q_S = -0.25$$

$$R_p := \left(1 + Q_S^2 \right) \cdot \text{Re}(Z_S) \qquad\qquad R_p = 212.5\,\text{ohm}$$

$$L_1 := \text{if}\left(Q_S = 0, 1000\text{H}, \frac{1 + Q_S^2}{Q_S^2} \cdot \frac{\text{Im}(Z_S)}{\omega} \right) \qquad L_1 = -150.313\,\text{nH}$$

With the source and load reactances deimbedded, we can synthesize the matching network using the real part of the source and the load. Our goal here is to size the L and the C such that source and the load sees the conjugate of it's impedance looking into the network. A matched network sliced anywhere in the network will have a conjugate impedance looking in either direction.

$$Z_{left} = \overline{Z_{right}}$$

The left side of the matching network consists of a parallel resistance and reactance. This can be expressed in a simplified form with the parallel resistance and the Q of the source.

$$Z_{left} = \cfrac{1}{\cfrac{1}{R_{left}} + \cfrac{1}{j \cdot L_{left}}} = \frac{1}{G_p + j \cdot B_{left}} = \frac{R_p}{\left(1 + j \cdot Q_{left} \right)}$$

The right side of the matching network consists of a series resistance and reactance. This can be expressed in a simplified form with the series resistance and the Q of the load.

$$\overline{Z_{right}} = R_{right} + \frac{1}{j \cdot C_{right}} = R_S + j \cdot X_{right} = R_S \cdot \left(1 - j \cdot Q_{right} \right)$$

Because the left and right resistances must be equal, and the reactances conjugates of each other, the Q's of the left and right sides must be equal. Setting the left impedance and right conjugate equal to each other, and setting the Q's equal gives the following equality.

$$\frac{R_p}{(1 + j \cdot Q)} = R_S \cdot (1 - j \cdot Q)$$

Solving this expression gives a general expression for the Q of matching network. Note the Q value is fixed by the source and load impedance for the L matching network. For higher order networks, and for equal source and load resistance, the Q of the network may be chosen.

$$Q := \sqrt{\frac{R_p}{R_S} - 1} \qquad Q = 1.803$$

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$$\sqrt{R_S}$$

This Q will serve as a lower bound for pi and T matching networks, and as an upper bound for cascaded L matching networks. Note the difference between network Q and component Q. Network Q is usually on the order of zero to three, to reduce sensitivity of the network. Component Q is much higher, typically 10 to 100, to

Given the Q of the left and right half sides of the network, and the effective source and load impedances, we can find the matching network inductance and capacitance without source inductance deembedded

$$C_S := \frac{1}{Q \cdot \omega \cdot R_S} \qquad C_S = 1.962\,\text{pF} \qquad L_p := \frac{R_p}{\omega \cdot Q} \qquad L_p = 20.845\,\text{nH}$$

The final impedance network is found by substracting the source inductance from the matching inductance and the load capacitance from the matching capacitance.

$$C := \text{if}\left(C_1 = C_S, 1000\text{F}, \frac{C_1 \cdot C_S}{C_1 - C_S} \right) \qquad \boxed{C = 1.536\,\text{pF}}$$

$$L := \text{if}\left(L_1 = L_p, 1000\text{H}, \frac{L_p \cdot L_1}{L_1 - L_p} \right) \qquad \boxed{L = 18.306\,\text{nH}}$$

This optimal power matching network corresponds to an input and output rms voltage swing of

$$V_i := I_i \cdot \frac{R_p}{2} \qquad V_i = 10.625\,\text{V}_{\text{rms}}$$

$$V_o := I_i \cdot \frac{|Z_L|}{2} \qquad V_o = 2.795\,\text{V}_{\text{rms}}$$

$$\text{Vi\_Ii}_{\text{opt}}(\omega) := \frac{1}{\dfrac{1}{Z_S} + \dfrac{1}{L \cdot j \cdot \omega} + \dfrac{1}{\left(Z_L + \dfrac{1}{j \cdot \omega \cdot C} \right)}} \qquad \left| \text{Vi\_Ii}_{\text{opt}}(\omega_c) \right| I_i = 10.625\,\text{V}_{\text{rms}}$$

$$\text{Vo\_Ii}_{\text{opt}}(\omega) := \frac{1}{\dfrac{1}{Z_S} + \dfrac{1}{L \cdot j \cdot \omega} + \dfrac{1}{\left(Z_L + \dfrac{1}{j \cdot \omega \cdot C} \right)}} \cdot \frac{Z_L}{Z_L + \dfrac{1}{j \cdot \omega \cdot C}} \qquad \left| \text{Vo\_Ii}_{\text{opt}}(\omega_c) \right| I_i = 5.762\,\text{V}_{\text{rms}}$$

Outputs

C = 1.536 pF   Load Capacitance (in series with load)
L = 18.306 nH   Source Inductance (in parallel with source)

Functions

Here's a function to calculate the matching network elements. It is a duplicate of procedure above.

$$\text{highL}(Z_S, Z_L, f) := \begin{vmatrix} \omega \leftarrow 2 \cdot \pi \cdot f \\[6pt] C_1 \leftarrow \text{if}\left(\text{Im}(Z_L) = 0, 1000\text{F}, \dfrac{-1}{\omega \cdot \text{Im}(Z_L)} \right) \\[6pt] R_S \leftarrow \text{Re}(Z_L) \\[6pt] Q_S \leftarrow \dfrac{\text{Im}(Z_S)}{\text{Re}(Z_S)} \\[6pt] R_p \leftarrow \left(1 + Q_S^2 \right) \cdot \text{Re}(Z_S) \\[6pt] L_1 \leftarrow \dfrac{1 + Q_S^2}{Q_S^2} \cdot \dfrac{\text{Im}(Z_S)}{\omega} \end{vmatrix}$$

$$Q \leftarrow \sqrt{\frac{R_p}{R_S} - 1}$$

$$L_p \leftarrow \frac{R_p}{\omega \cdot Q}$$

$$C_S \leftarrow \frac{1}{Q \cdot \omega \cdot R_S}$$

$$C \leftarrow \text{if}\left(C_1 = C_S, 1000F, \frac{C_1 \cdot C_S}{C_1 - C_S}\right)$$

$$L \leftarrow \text{if}\left(L_1 = L_p, 1000H, \frac{L_p \cdot L_1}{L_1 - L_p}\right)$$

$$\begin{pmatrix} \dfrac{L}{H} \\ \dfrac{C}{F} \end{pmatrix}$$

Examples

$$x := \text{highL}\left(Z_S, Z_L, f\right)$$

$$L := x_1 \cdot H \qquad L = 18.306 \, nH$$

$$C := x_2 \cdot F \qquad C = 1.536 \, pF$$

$$Z_i := (j \cdot \omega \cdot L) \parallel \left(Z_L + \frac{1}{j \cdot \omega \cdot C} \right) \qquad Z_i = 200 + 50i \, \text{ohm} \qquad \text{Ideal Input impedance at desired frequency}$$

$$Z_o := Z_S \parallel (j \cdot \omega \cdot L) + \frac{1}{j \cdot \omega \cdot C} \qquad Z_o = 50 - 25i \, \text{ohm} \qquad \text{Ideal Output impedance at desired frequency}$$

S Parameters

S-parameter is an abbreviation for scattering parameters. S-parameters are a measure of the power gain of a network. The term scattering comes from the concept of a cue ball scattering other balls as it transfers power to them. S_{ij} is the measure of power gain from port j to port i. Specifically, the square root of power gain. For example, S_{21}, the most useful S-parameter, is the measure of the ratio of output power to the available input power. This is an important sentence. This sentence is used to perform hand calculations. The output power is easy to explain, it is V_{Orms}^2/R_L. The available input power is trickier to explain. Available input power is the maximum power that can be delivered from a source. For a source impedance of R_S, and a source voltage of V_{Srms}; the available input power is $(V_{Srms}/2)^2/R_S$. Why, because maximum power is delivered is to a resistance of R_S. Thus a voltage division of two for the voltage, when delivering maximum available power. Thus the power gain of a network, S_{21}, is found by dividing the two powers $2*V_o/V_S*sqrt(R_S/R_L)$.

S-parameters are most conveniently calculated by converting an impedance or admittance matrix into a S-parameter matrix. So we start by finding the impedance (Z) parameters for the highpass L matching network. Z parameters are found by driving a port with a current source, leaving the rest of the ports open, and measuring the voltages at all the ports.

$$Z(\omega) := \begin{pmatrix} j \cdot \omega \cdot L & j \cdot \omega \cdot L \\ j \cdot \omega \cdot L & \dfrac{1}{j \cdot \omega \cdot C} + j \cdot \omega \cdot L \end{pmatrix}$$

Admittance (Y) parameters are found by driving a port with a voltage source, shorting the rest, and measuring the current at all the ports. The admittance matrix is the inverse of the Z matrix.

$$Y(\omega) := Z(\omega)^{-1}$$

Y parameters are converted to a S parameters with a common impedance on all ports using this expression from "Microwave Engineering" by Pozar.

$$Y2S(Y, Y_0) := \left| \begin{array}{l} I \leftarrow Y_0 \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \\ (I - Y) \cdot (I + Y)^{-1} \end{array} \right.$$

This scattering parameter conversion routine (to convert to arbitrary source and load impedances) is given "Applied RF Techniques I" lecture notes, but is incorrect. A correct version of the routine is found on page 31 of "Microwave Amplifiers and Oscillators," by Christian Gentili.

$$S_{conv}(S, Z_{Sbegin}, Z_{Send}, Z_{Lbegin}, Z_{Lend}) := \left| \begin{array}{l} \Gamma_S \leftarrow \dfrac{Z_{Send} - Z_{Sbegin}}{Z_{Send} + Z_{Sbegin}} \\[2em] \Gamma_L \leftarrow \dfrac{Z_{Lend} - Z_{Lbegin}}{Z_{Lend} + Z_{Lbegin}} \\[2em] D \leftarrow (1 - \Gamma_S \cdot S_{1,1}) \cdot (1 - \Gamma_L \cdot S_{2,2}) - \Gamma_S \cdot \Gamma_L \cdot S_{1,2} \cdot S_{2,1} \\[2em] A_1 \leftarrow \dfrac{1 - \overline{\Gamma_S}}{|1 - \Gamma_S|} \cdot \sqrt{1 - (|\Gamma_S|)^2} \\[2em] A_2 \leftarrow \dfrac{1 - \overline{\Gamma_L}}{|1 - \Gamma_L|} \cdot \sqrt{1 - (|\Gamma_L|)^2} \end{array} \right.$$

$$\left|\left[\begin{array}{cc} \dfrac{\overline{A_1}}{A_1}\cdot\dfrac{\left(1-\Gamma_L\cdot S_{2,2}\right)\cdot\left(S_{1,1}-\overline{\Gamma_S}\right)+\Gamma_L\cdot S_{1,2}\cdot S_{2,1}}{D} & \dfrac{\overline{A_2}}{A_1}\cdot\dfrac{S_{1,2}\cdot\left[1-\left(\left|\Gamma_S\right|\right)\right.}{D} \\[3mm] \dfrac{\overline{A_1}}{A_2}\cdot S_{2,1}\cdot\dfrac{1-\left(\left|\Gamma_L\right|\right)^2}{D} & \dfrac{\overline{A_2}}{A_2}\cdot\dfrac{\left(1-\Gamma_S\cdot S_{1,1}\right)\cdot\left(S_{2,2}-\overline{\Gamma_L}\right)\cdot}{D} \end{array}\right.\right.$$

$Z_0 := 50\Omega$ Reference Impedanace for S-Parameters

$S_{50}(\omega) := \text{Y2S}\left(Y(\omega), \dfrac{1}{Z_0}\right)$ S-parameters of ideal matching network with actual load and source impedances.

$S(\omega) := S_{conv}\left(S_{50}(\omega), Z_0, Z_S, Z_0, Z_L\right)$ 50 Ohm S-parameters of ideal matching network.

       Plots of lossy and ideal S-paramaters of the matching network verses frequency. Be careful when using these plots, as they do not reflect the change in source and load impedance vs. frequency (for example if driver output impedance is capacitive).

$numi := 200$ Number of points in frequency plot

$i := 1.. numi$

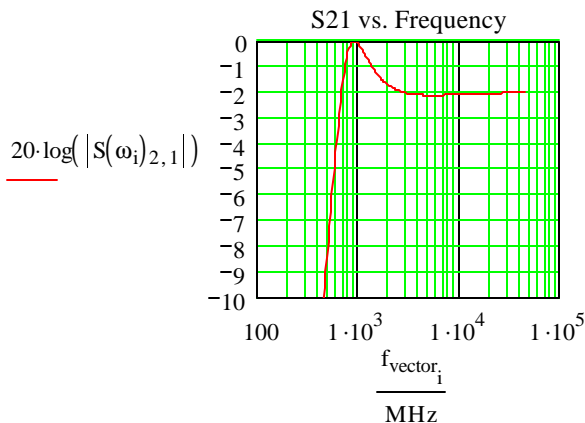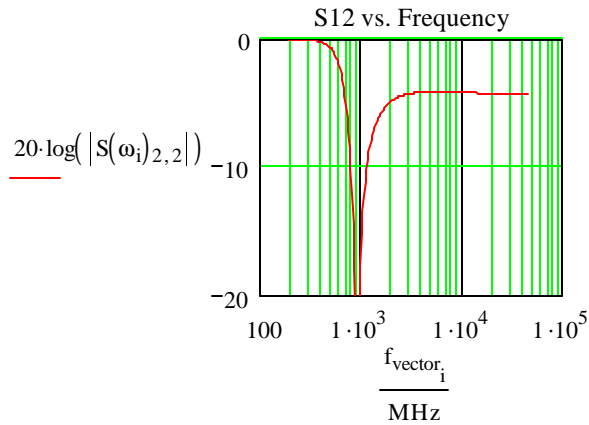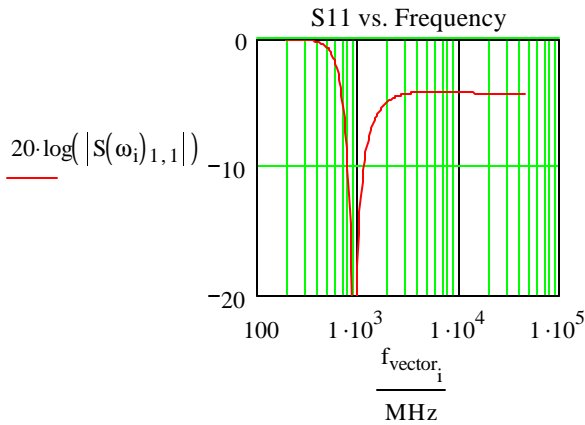$f_{start} := \dfrac{f}{5}$ $f_{stop} := f\cdot 50$ Starting and stopping frequencies for plot

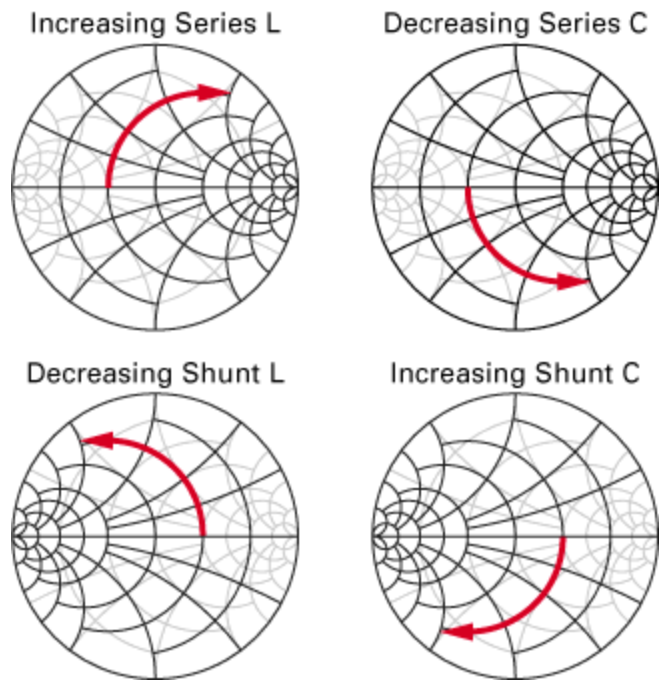$f_{vector_i} := f_{start}\cdot\left(\dfrac{f_{stop}}{f_{start}}\right)^{\frac{i}{numi}}$ $\omega_i := 2\cdot\pi\cdot f_{vector_i}$ $s_i := j\cdot\omega_i$

S11 vs. Frequency

$20\cdot\log\left(\left|S(\omega_i)_{1,1}\right|\right)$

S12 vs. Frequency

$20\cdot\log\left(\left|S(\omega_i)_{2,2}\right|\right)$

S21 vs. Frequency

$20\cdot\log\left(\left|S(\omega_i)_{2,1}\right|\right)$

S22 vs. Frequency

$20\cdot\log\left(\left|S(\omega_i)_{1,2}\right|\right)$

▶ Grid

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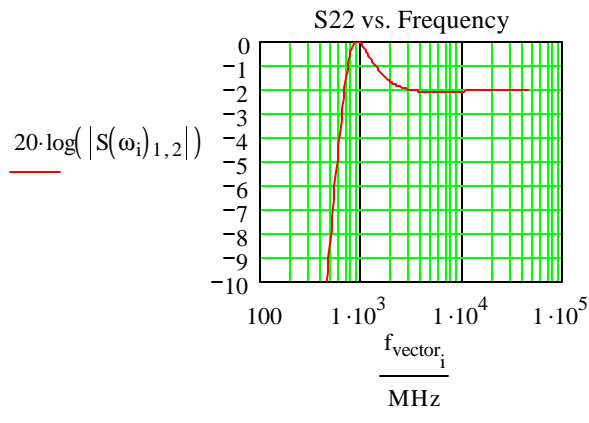
Impedance Matching with a Smith Chart

Increasing Series L

Decreasing Series C

Decreasing Shunt L

Increasing Shunt C

I am personally not a big fan of smith charts, but they have some historical value, are commonly used in network analyzers, and they allow you to plot infinite change in impedance vs. infinite frequency in a finite area. Here we describe a common older method for graphically matching a complex source impedance to a complex load impedance is using a Smith chart.

Step 1: The source and load impedances are converted to reflection coefficients and placed on the smith chart. One of the two impedances is conjugated, so that a conjugate match is achieved. Here we arbitrarily choose the source to be the conjugated impedance.

$$\Gamma_S := \frac{Z_S - Z_0}{Z_S + Z_0} \qquad \left|\Gamma_S\right| = 0.62 \qquad \angle\Gamma_S = -7.125° \qquad\qquad \Gamma_L := \frac{\overline{Z_L} - Z_0}{\overline{Z_L} + Z_0} \qquad \left|\Gamma_L\right| = 0.243 \qquad \angle\Gamma_L = -75.964°$$

Step 2: Using a protractor, with an origin at the left side of the chart, a circle is drawn which includes the source point. This circuit is represents the sweeping of inductance from an infinite parallel inductance to the proper value for matching.

$$L_{vector_i} := \frac{L \cdot numi}{i}$$

$$Z_{left_i} := Z_S \| \left(j \cdot \omega_c \cdot L_{vector_i}\right) \qquad\qquad \Gamma_{left_i} := \frac{Z_{left_i} - Z_0}{Z_{left_i} + Z_0}$$

GridZ
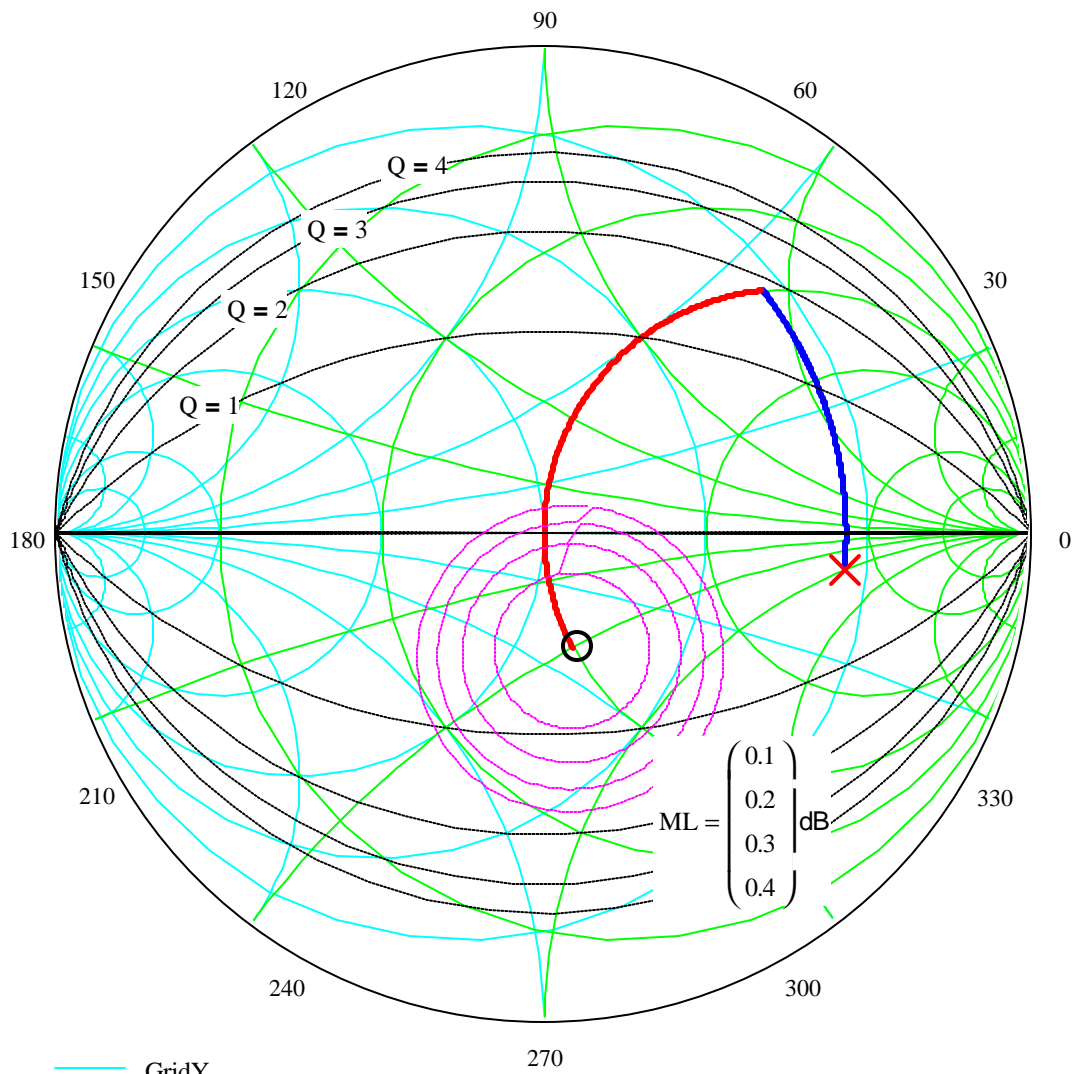Sweeping L
✕✕ Source Impedance
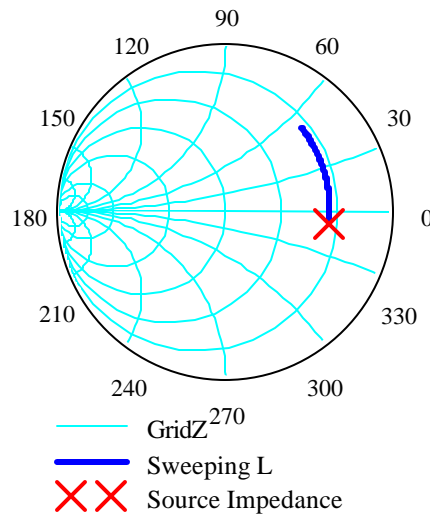
Step 3: Using a protractor, with an origin at the right side of the chart, a circle is drawn which includes the load point. This circuit is represents the sweeping of inductance from an infinite parallel inductance to the proper value for matching.

$$C_{vector_i} := \frac{numi}{i} \cdot C$$

$$Z_{right_i} := Z_S \| \left(j \cdot \omega_c \cdot L\right) + \frac{1}{j \cdot \omega_c \cdot C_{vector_i}} \qquad\qquad \Gamma_{right_i} := \frac{Z_{right_i} - Z_0}{Z_{right_i} + Z_0}$$

▶ Q and VSWR contours

Smith chart showing impedance matching. Labeled angular markings: 90, 120, 60, 150, 30, 180, 0, 210, 330, 240, 300, 270.

Q = 4
Q = 3
Q = 2
Q = 1

$$ML = \begin{pmatrix} 0.1 \\ 0.2 \\ 0.3 \\ 0.4 \end{pmatrix} dB$$

Legend:
- GridY
- GridZ
- Sweeping L
- Sweeping C
- × × Source Impedance
- ○ ○ Conjugate Load Impedance
- Constant Q Circles
- Constant VSWR Circles

S-Parameters on Smith Chart Plots

Here is a plot of the S parameters for the highpass L matching network vs. frequency on the Smith chart. Ugh, I find this useless, unless you narrow the bandwidth just to the frequency of interest, which hides the filtering properties.
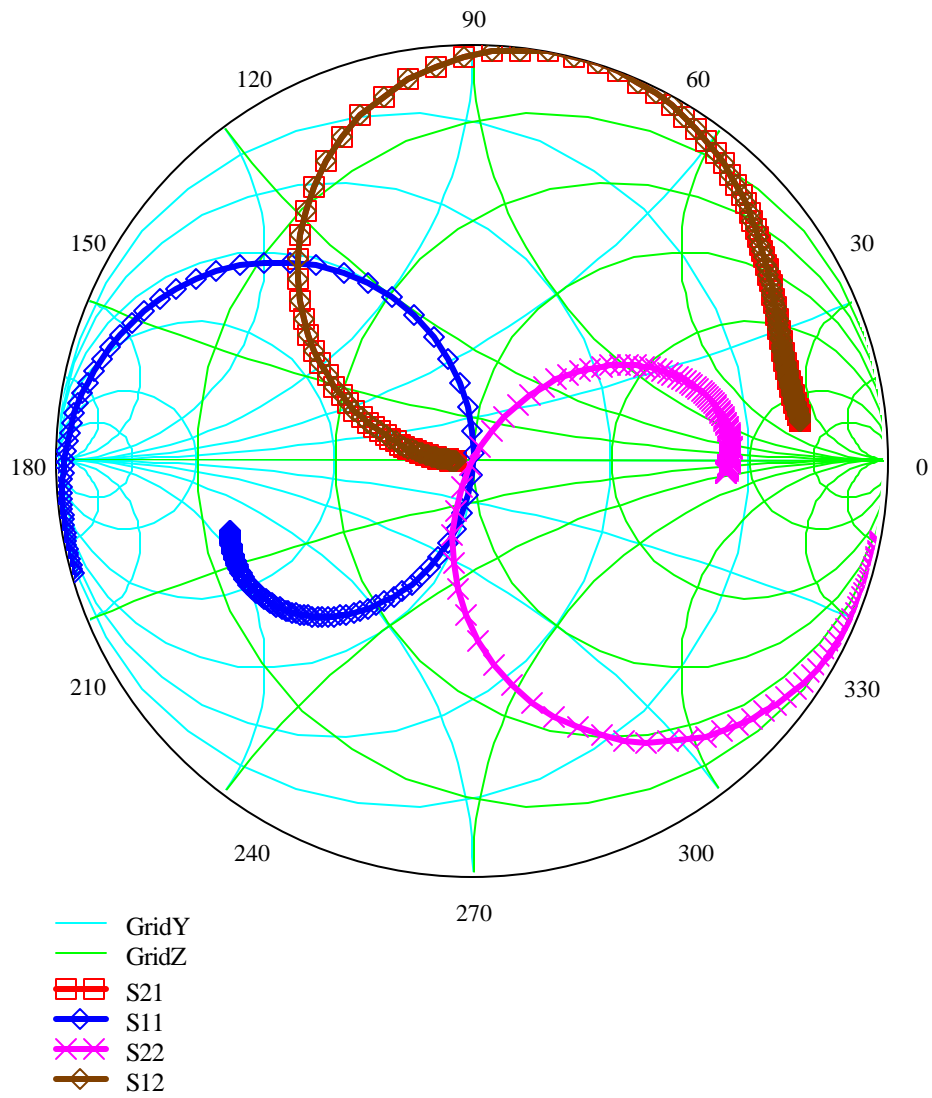
Sensitivity Analysis

All elements of a real circuit have process variations, which affects the performance of the circuit. The following calculations calculate loss due due to these process variations. The calculation is initially performed with purely resistive source and load impedances. For purely passive networks with small variations, the S_{11} and S_{22} sensitivities are the same, as are the S_{21} and S_{12} sensitivities. Two first order the sensitivity of S_{21} and S_{11} to process and center frequencies is zero.

$$Q := \sqrt{\frac{Re(Z_S)}{Re(Z_L)} - 1} \qquad Q = 1.732$$

$$C := \frac{1}{Q \cdot \omega_c \cdot Re(Z_L)} \qquad C = 2.042\,pF$$

$$L := \frac{Re(Z_S)}{\omega_c \cdot Q} \qquad L = 20.42\,nH$$

$\Delta\omega\_\omega := 0 \qquad \Delta RS\_RS := 0 \qquad \Delta C\_C := 0 \qquad \Delta L\_L := 0 \qquad \Delta RL\_RL := 0$

$$\omega_c := \omega_c \cdot (1 + \Delta\omega\_\omega)$$

$$L := L \cdot (1 + \Delta L\_L) \qquad C := C \cdot (1 + \Delta C\_C)$$

$$Re(Z_L) := Re(Z_L) \cdot (1 + \Delta RL\_RL)$$

$$Re(Z_S) := Re(Z_S) \cdot (1 + \Delta RS\_RS)$$

$$S_{21c}(\omega_c) := \frac{\omega_c^2 \cdot L \cdot C \cdot 2 \cdot \sqrt{Re(Z_L) \cdot Re(Z_S)}}{\sqrt{\omega_c^2 \cdot (Re(Z_L) \cdot C \cdot Re(Z_S) + L)^2 + \left[Re(Z_S) + -(Re(Z_S) + Re(Z_L)) \cdot \omega_c^2 \cdot C \cdot L \right]^2}}$$
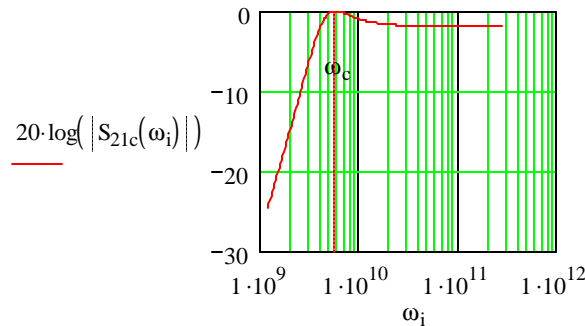
$$Z_{in}(\omega) := \frac{j \cdot \omega \cdot L \cdot \left(Re(Z_L) + \frac{1}{j \cdot \omega \cdot C} \right)}{j \cdot \omega \cdot L + Re(Z_L) + \frac{1}{j \cdot \omega \cdot C}} \qquad Z_{in}(\omega_c) = 200\,\Omega$$

$$S_{11}(\omega) := \left| \frac{Z_{in}(\omega) - Re(Z_S)}{Z_{in}(\omega) + Re(Z_S)} \right| \qquad S_{11}(\omega_c) = 0$$

$$S_{11} = \frac{Q}{2} \cdot \Delta L\_L + \frac{Q}{2} \cdot \Delta C\_C + \Delta\omega\_\omega \frac{Q^2}{\sqrt{Q^2 + 1}} + \frac{\Delta RL\_RL \cdot Q}{2} + \frac{\Delta RS\_RS \cdot Q}{2}$$

$$\Delta S21\_S21 = \frac{-1}{8} \cdot Q^2 \cdot \Delta L\_L^2 - \frac{1}{8} \cdot Q^2 \cdot \Delta C\_C^2 - \frac{Q^4}{2 \cdot (Q^2 + 1)} \cdot \Delta\omega\_\omega^2 - \frac{1}{8} \cdot \Delta RS\_RS^2 - \frac{1}{8} \cdot \Delta RL\_RL^2$$

First order sensitivity to process variations is zero

$20 \cdot \log(|S_{21c}(\omega_i)|)$

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Amplitude Constrained Impedance Transformation

Often a matching network of this nature are designed with a given input power and has constraints on either the input node or the output node voltage swings. This situation is present in every receiver typically at internal interfaces, such as between an LNA and a mixer, between a VCO and the mixer, and between intermediate stages of a power amplifier chain. These voltage limitations become more evident with process advances, where impedances become high relative to 50 ohms, and supply voltages are reduced to prevent device damage.

Given a constrained input swing, the output swing should be maximized.

$$V_o = V_{imax} \cdot \left| \frac{Z_L}{\frac{1}{j \cdot \omega \cdot C} + Z_L} \right| = V_{imax} \cdot \frac{R_L^2 + X_L^2}{R_L^2 + \left(X_L - \frac{1}{\omega \cdot C} \right)^2}$$

$C := 0.859 \text{pF}$

$L := 22.344 \text{nH}$

The output is maximized under contrained input swing when the matching capacitance resonates with the reactance of the load.

$$\boxed{C := \frac{1}{\omega_c \cdot Im(Z_L)}} \quad C = 7.074 \, pF$$

Thus the output voltage is constrained to be

$$V_{omax} = V_{imax} \cdot \left(1 + Q_L^2 \right)$$

$V_i = 10.625 \, V$

This capacitance corresponds to an input swing of

$$V_i = I_i \cdot \left| \frac{1}{G_S + \frac{1}{Re(Z_L)} + j \cdot \left(\omega_c \cdot C_S + \frac{-1}{\omega_c \cdot L} \right)} \right| = I_i \cdot \sqrt{\frac{1}{\left(G_S + \frac{1}{Re(Z_L)} \right)^2 + \left(\omega_c \cdot C_S - \frac{1}{\omega_c \cdot L} \right)^2}}$$

where G_S and C_S are the parallel equivalent input impedance $G_S := \frac{1}{Re(Z_S) \cdot \left(1 + Q_S^2 \right)}$ and $C_S := \frac{-Q_S^2}{\omega_c \cdot Im(Z_S) \cdot \left(1 + Q_S^2 \right)}$

Solving for the inductance gives two possible solutions, we choose the one with real inductance values:

$$\boxed{L := \frac{1}{\omega_c^2 \cdot C_S - \omega_c \cdot \sqrt{\left(\frac{I_i}{V_{imax}} \right)^2 - \left(G_S + \frac{1}{Re(Z_L)} \right)^2}}}$$

$L = -66.806 \, nH$

$$\boxed{L := \frac{1}{\omega_c^2 \cdot C_S + \omega_c \cdot \sqrt{\left(\frac{I_i}{V_{imax}} \right)^2 - \left(G_S + \frac{1}{Re(Z_L)} \right)^2}}}$$

$L = 35.368 \, nH$

Note that this expression is only valid for small Vinmax constraints: $V_{imax} < \frac{I_i}{G_S + \frac{1}{Re(Z_L)}}$ $\quad \frac{I_i}{G_S + \frac{1}{Re(Z_L)}} = 4.048 \, V$
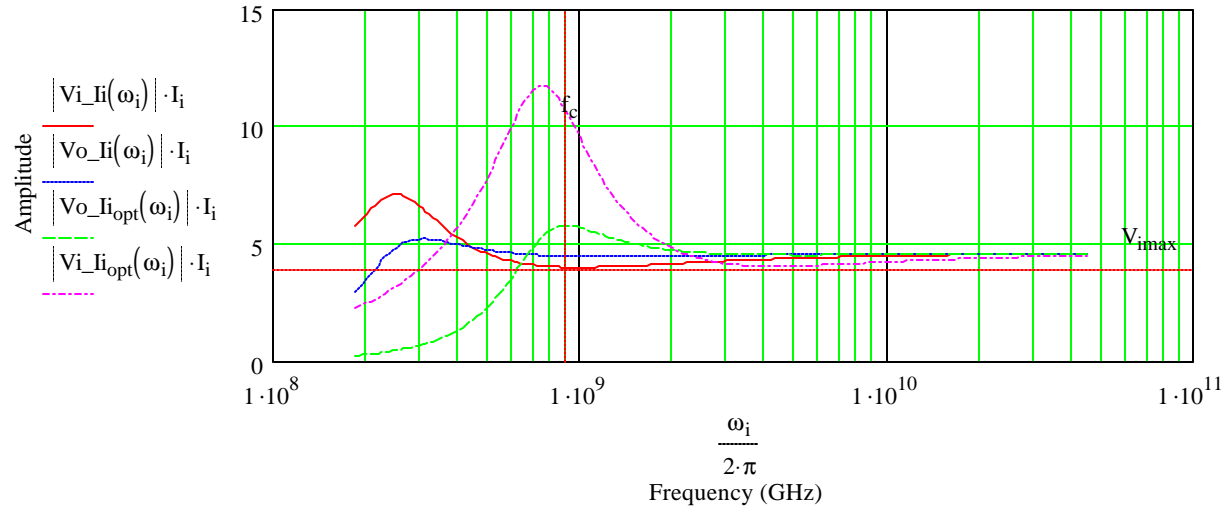
$Z_i := \left(j \cdot \omega_c \cdot L \right) \| \left(Z_L + \frac{1}{j \cdot \omega_c \cdot C} \right)$ $\quad Z_i = 47.059 + 11.765i \, ohm$

$Z_o := Z_S \| \left(j \cdot \omega_c \cdot L \right) + \frac{1}{j \cdot \omega_c \cdot C}$ $\quad Z_o = 128 + 79i \, ohm$

$Vi\_Ii(\omega) := \frac{1}{\frac{1}{\quad} \cdot \frac{1}{\quad} \cdot \frac{1}{\quad}}$ $\quad\quad\quad \left| Vi\_Ii(\omega_c) \right| I_i = 4 \, V_{rms}$

$$\text{Vo\_Ii}(\omega) := \cfrac{\cfrac{1}{\overline{Z_S}} + \cfrac{1}{\overline{L \cdot j \cdot \omega}} + \cfrac{1}{\overline{\left(Z_L + \cfrac{1}{j \cdot \omega \cdot C}\right)}}}{\cfrac{1}{Z_S} + \cfrac{1}{L \cdot j \cdot \omega} + \cfrac{1}{\left(Z_L + \cfrac{1}{j \cdot \omega \cdot C}\right)}} \cdot \cfrac{Z_L}{Z_L + \cfrac{1}{j \cdot \omega \cdot C}}$$

$\left| \text{Vo\_Ii}(\omega_c) \right| I_i = 4.472 \, V_{rms}$

Legend:
$\left| \text{Vi\_Ii}(\omega_i) \right| \cdot I_i$
$\left| \text{Vo\_Ii}(\omega_i) \right| \cdot I_i$
$\left| \text{Vo\_Ii}_{opt}(\omega_i) \right| \cdot I_i$
$\left| \text{Vi\_Ii}_{opt}(\omega_i) \right| \cdot I_i$

Amplitude

$\dfrac{\omega_i}{2 \cdot \pi}$

Frequency (GHz)

15

Layout Notes for the Highpass L Matching Network

   The figure at the beginning of this report shows a single inductor placed across the source impedance use to resonate out it's reactance. In real life, there is no such thing as a two-terminal inductance, as inductance is a measure of reactance around a loop. In the figure, the two-terminal inductance represents a partial inductance. The word "partial" is used, because the inductance is only part of a loop. So where is the rest of the loop?

   In this circuit there are actually two loops (assuming the source current and resistance are a single lumped element): A source loop and a load loop. In the absence of the matching inductor a single loop exists between the source and load. The matching inductor acts as a shunt in the middle of the loop. The number of loops a circuit has can be easily seen, and is the same number as the number of equations for circuit analysis using KVL. The number of inductors in a circuit is equal to the number of loops, with mutual inductances between every inductance and every o ther inductance. In a simplified form, the mutual inductances between touching loops may be omitted. Thus the complete matching network is shown in the following figure. Note that the ubiquitous "ground" terminal is placed on only one node as in real life only one point in space can truely be called ground, and in the end it doesn't matter which point that is. The partial inductances are distributed around the entire loop, so it is inappropriate to think of the source ground and the load ground as the same point.
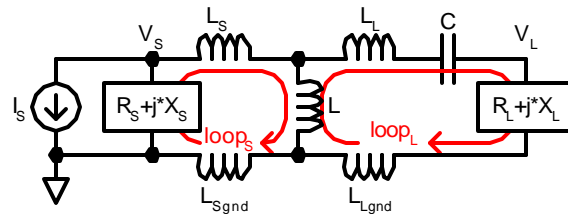

Fig. 1: Highpass L matching network with all partial inductances.

   When designing a matching network it is important to add the partial inductances of the source and load loop to the source and load impedances. It is extremely common to design an RF circuit and then find out in testing that the power is matched for a lower frequency, because partial inductances were neglected.
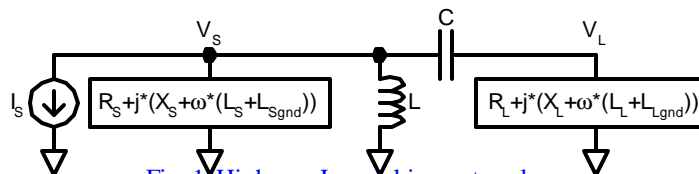

Fig. 1: Highpass L matching network

   These parasitic inductances can be minimized by routing the signal path from the matching network to the load very close to the ground return path from the load. The same is true for the routing of the source to the matching network.

   Also to be included in the matching network design is the deimbedding of the parasitic capacitances of the matching elements themselves.

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