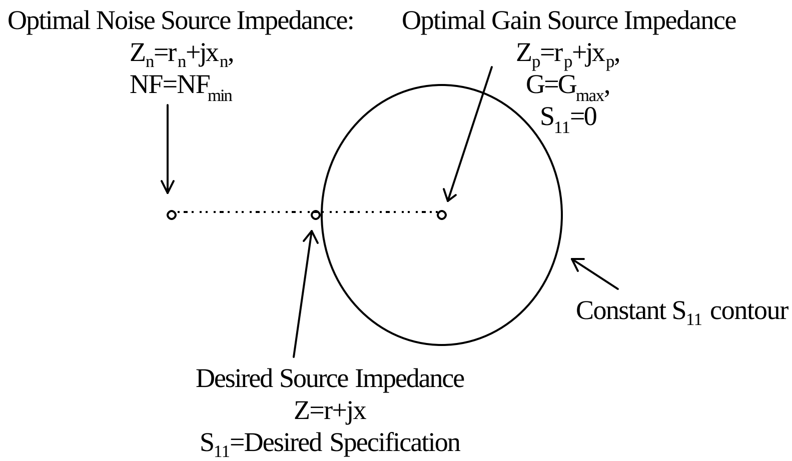
Optimal Source Impedance to Trade-Off Power and Noise and to Achieve a Desired S11



▶ useful functions and identities
▶ Units
▶ Constants

Table of Contents

I. **Introduction**
IV. **Inputs**
IX. **Copyright and Trademark Notice**

Introduction

In the design of a low noise amplifier,LNA, or mixer driver stage there exists an optimal source source impedance to provide the maximum power transfer into the transistor and thus the maximum gain. There also exists an optimal source impedance, Z_{nopt}, to provide the minimum noise figure for the circuit. At the maximum power source impedance, Z_{popt}, the input is matched to the source impedance and no reflections occur. This is desirable, because the filters attached to designed for the LNA or mixer, are designed for a matched load impedance, but can tolerate a certain amount of deviation from it's desired impedance. The amount of acceptable deviation is usually specified by the maximum return loss, or S_{11}, the filter can handle. The routine below finds the source impedance to the LNA or mixer, which will provide the lowest noise figure and mximum gain, while maintaining a desired S11 specification.

The calculation is performed by drawing a straight line between the optimal noise and power source impedances and finding where it intersects the desired S11 contour. This line intersects the contour at two points, so the point must be chosen, which is closet to the optimal noise source impedance. Sometimes, the desired S11 specification is acheived at the optimal noise source impedance. In this case the optimal noise source impedance is chosen over the intersecting point. This calculation is not optimal in the truest sense, because the noise and power gain circles are not coencentric, but it serves as an excellent estimate to meet design criteria.

A few important notes: 1. The optimal source impedance for minimum noise figure should be calculated with estimates of ALL back-end noise components (mixers, PGA's, filters, A/Ds). Doing so will push the noise match much closer to the power match. 2. Simultaneous input-output match of a device (i.e optimum power match) may be unstable, which will push the desired source impedance closer to the noise match.

Derivation

$Z_S = r + j\cdot x \qquad r = \mathrm{Re}(Z_S) \qquad x = \mathrm{Im}(Z_S)$ Desired Source Impedane

$Z_n = r_n + j\cdot x_n \qquad r_n = \mathrm{Re}(Z_n) \qquad x_n = \mathrm{Im}(Z_n)$ Optimal Noise Source Impedance

$Z_p = r_p + j\cdot x_p \qquad r_p = \mathrm{Re}(Z_p) \qquad x_p = \mathrm{Im}(Z_p)$ Optimal Power Source Impedance

$Z_{in} = \overline{Z_p} \qquad r_i = \mathrm{Re}(Z_{in}) \qquad x_i = \mathrm{Im}(Z_{in})$ Input Impedance

$$m = \frac{x_p - x_n}{r_p - r_n}$$

Slope of line between Optimal Noise and Power Impedance

$$x = \frac{x_p - x_n}{r_p - r_n}\cdot(r - r_n) + x_n \qquad x = m\cdot(r - r_n) + x_n$$

Equation Number 1: Straight line between noise and power impedances.

$$S_{11} = \left|\frac{Z_{in} - \overline{Z_S}}{Z_{in} + Z_S}\right| \qquad S_{11} = 10^{\frac{S_{11dB}}{20}}$$

Input Match Equation

$$S_{11dB} = 20\cdot\log\left(\left|\frac{r_i + j\cdot x_i - r + j\cdot x}{r_i + j\cdot x_i + r + j\cdot x}\right|\right)$$

$$S_{11}^2\cdot(r_i + r)^2 = (r_i - r)^2 + (x_i + x)^2\cdot\left(1 - S_{11}^2\right)$$

$$0 = r_i^2 - 2\cdot r_i\cdot r\cdot\left(\frac{1 + S_{11}^2}{1 - S_{11}^2}\right) + r^2 + (x_i + x)^2$$

Equation Number 2

$$g = \frac{1 + S_{11}^2}{1 - S_{11}^2}$$

Inserting equation number of x into equation number two for x

$$0 = r_i^2 - 2\cdot r_i\cdot r\cdot g + r^2 + \left[x_i + m\cdot(r - r_n) + x_n\right]^2$$

$$0 = \left(1 + m^2\right)\cdot r^2 + \left[-2\cdot r_i\cdot g + 2\cdot(x_i - m\cdot r_n + x_n)\cdot m\right]\cdot r + r_i^2 + (x_i - m\cdot r_n + x_n)^2$$

$$a = 1 + m^2 \qquad b = 2\cdot(x_i + x_n - m\cdot r_n)\cdot m - 2\cdot r_i\cdot g \qquad c = r_i^2 + (x_i + x_n - m\cdot r_n)^2$$

$$0 = a\cdot r^2 + b\cdot r + c$$

$$r = \begin{bmatrix} \frac{-1}{(2\cdot a)}\cdot\left(b + \sqrt{b^2 - 4\cdot a\cdot c}\right) \\ \frac{-1}{(2\cdot a)}\cdot\left(b - \sqrt{b^2 - 4\cdot a\cdot c}\right) \end{bmatrix} \qquad x = m\cdot(r - r_n) + x_n$$

Inputs

$S_{11goaldB} := -10dB$ Desired S_{11} value

$Z_p := (21 + 10\cdot j)ohm$ Optimal Source Impedance for Maximum Gain

$Z_n := (10 + j\cdot 10)ohm$ Optimal Source Impedance for Minimum Noise Figure

Calculation

$$S_{11goal} := 10^{\frac{S_{11goaldB}}{20}} \qquad S_{11goal} = 0.316$$

$$Z_{in} := \overline{Z_p} \qquad Z_{in} = 21 - 10i\,ohm$$

$$r_i := Re(Z_{in}) \qquad x_i := Im(Z_{in}) \qquad r_i = 21\,ohm \qquad x_i = -10\,ohm$$

$$r_p := Re(Z_p) \qquad x_p := Im(Z_p) \qquad r_p = 21\,ohm \qquad x_p = 10\,ohm$$

$$r_n := Re(Z_n) \qquad x_n := Im(Z_n) \qquad r_n = 10\,ohm \qquad x_n = 10\,ohm$$

$$S_{11n} := 20 \cdot \log\left(\left|\frac{Z_{in} - \overline{Z_n}}{Z_{in} + Z_n}\right|\right) \qquad S_{11n} = -8.999$$

$$g := \frac{1 + S_{11goal}^2}{1 - S_{11goal}^2} \qquad g = 9.807\,ms^{-2}$$

$$m := \frac{x_p - x_n}{r_p - r_n} \qquad m = 1\,m$$

$$a := 1 + m^2 \qquad a = 1$$

$$b := 2 \cdot (x_i + x_n - m \cdot r_n) \cdot m - 2 \cdot r_i \cdot g \qquad b = -51.333\,ohm$$

$$c := r_i^2 + (x_i + x_n - m \cdot r_n)^2 \qquad c = 441\,ohm^2$$

$$r_1 := \frac{-1}{(2 \cdot a)} \cdot \left(b + \sqrt{b^2 - 4 \cdot a \cdot c}\right) \qquad r_1 = 10.909\,ohm$$

$$x_1 := m \cdot (r_1 - r_n) + x_n \qquad x_1 = 10\,ohm$$

$$d_1 := (r_1 - r_n)^2 + (x_1 - x_n)^2 \qquad \sqrt{d_1} = 0.909\,ohm$$

$$r_2 := \frac{-1}{(2 \cdot a)} \cdot \left(b - \sqrt{b^2 - 4 \cdot a \cdot c}\right) \qquad r_2 = 40.424\,ohm$$

$$x_2 := m \cdot (r_2 - r_n) + x_n \qquad x_2 = 10\,ohm$$

$$d_2 := (r_2 - r_n)^2 + (x_2 - x_n)^2 \qquad \sqrt{d_2} = 30.424\,ohm$$

$$r := if(d_1 < d_2, r_1, r_2) \qquad r = 10.909\,ohm$$

$$x := if(d_1 < d_2, x_1, x_2) \qquad x = 10\,ohm$$

$$Z_S := if\left(S_{11n} < S_{11goaldB}, Z_n, r + \sqrt{-1} \cdot x\right) \qquad Z_S = 10.909 + 10i\,ohm$$

Outputs

$$Z_S = 10.909 + 10i\,ohm$$

Check Results:

$$S_{11} := 20 \cdot \log\left(\left|\frac{Z_{in} - \overline{Z_S}}{Z_{in} + Z_S}\right|\right) \qquad S_{11} = -10\,dB$$

Function

$$Z_{Ss11}(Z_p, Z_n, S_{11goaldB}) := \left| \begin{array}{l}
S_{11goal} \leftarrow 10^{\frac{S_{11goaldB}}{20}} \\
Z_{in} \leftarrow \overline{Z_p} \\
r_i \leftarrow \mathrm{Re}(Z_{in}) \\
x_i \leftarrow \mathrm{Im}(Z_{in}) \\
r_p \leftarrow \mathrm{Re}(Z_p) \\
x_p \leftarrow \mathrm{Im}(Z_p) \\
r_n \leftarrow \mathrm{Re}(Z_n) \\
x_n \leftarrow \mathrm{Im}(Z_n) \\
S_{11n} \leftarrow 20 \cdot \log\left(\left| \frac{Z_{in} - \overline{Z_n}}{Z_{in} + Z_n} \right| \right) \\
g \leftarrow \frac{1 + S_{11goal}^2}{1 - S_{11goal}^2} \\
m \leftarrow \frac{x_p - x_n}{r_p - r_n} \\
a \leftarrow 1 + m^2 \\
b \leftarrow 2 \cdot (x_i + x_n - m \cdot r_n) \cdot m - 2 \cdot r_i \cdot g \\
c \leftarrow r_i^2 + (x_i + x_n - m \cdot r_n)^2 \\
r_1 \leftarrow \frac{-1}{(2 \cdot a)} \cdot \left(b + \sqrt{b^2 - 4 \cdot a \cdot c} \right) \\
x_1 \leftarrow m \cdot (r_1 - r_n) + x_n \\
d_1 \leftarrow (r_1 - r_n)^2 + (x_1 - x_n)^2 \\
r_2 \leftarrow \frac{-1}{(2 \cdot a)} \cdot \left(b - \sqrt{b^2 - 4 \cdot a \cdot c} \right) \\
x_2 \leftarrow m \cdot (r_2 - r_n) + x_n \\
d_2 \leftarrow (r_2 - r_n)^2 + (x_2 - x_n)^2 \\
r \leftarrow \mathrm{if}(d_1 < d_2, r_1, r_2) \\
x \leftarrow \mathrm{if}(d_1 < d_2, x_1, x_2) \\
Z_{Sans} \leftarrow \mathrm{if}\left(S_{11n} < S_{11goaldB}, Z_n, r + \sqrt{-1} \cdot x\right) \\
Z_{Sans}
\end{array} \right.$$

Example

$S_{11goaldB} := -15dB$

$Z_p := (50 + j\cdot 10)ohm$ $\qquad G_{max} := 15dB$

$Z_n := (90 + j\cdot 10)ohm$ $\qquad NF_{min} := 1.1dB$

$Z_{in} := \overline{Z_p}$ $\qquad Z_{in} = 50 - 10i\,ohm$

$Z_S := Z_{Ss11}(Z_p, Z_n, S_{11goaldB})$ $\qquad Z_S = 71.629 + 10i\,ohm$

$$S_{11} := 20\cdot \log\left(\left|\frac{Z_{in} - \overline{Z_S}}{Z_{in} + Z_S}\right|\right) \qquad S_{11} = -15\,dB$$

$\text{if}\left(S_{11n} < S_{11goaldB}, Z_n, r + \sqrt{-1}\cdot x\right) = 10.909 + 10i\,ohm$

Plots

$num := 100$

$i := 1 .. num$

$$R_{Save} := \frac{Re(Z_n) + Re(Z_p)}{2}$$

$$R_{Sstart} := \frac{R_{Save}}{5} \qquad R_{Sstop} := R_{Save}\cdot 5$$

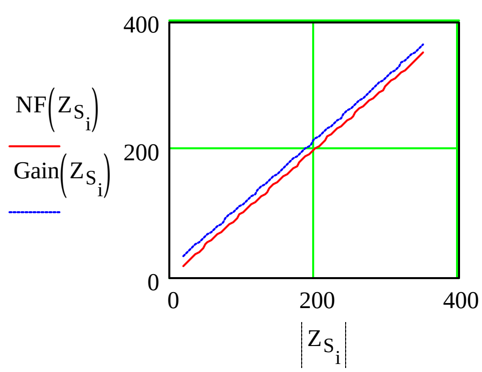
$$R_{S_i} := R_{Sstart} + \frac{i-1}{num-1}\cdot\left(R_{Sstop} - R_{Sstart}\right)$$

$$X_{S_i} := \frac{Im(Z_n) - Im(Z_p)}{Re(Z_n) - Re(Z_p)}\cdot\left(R_{S_i} - Re(Z_p)\right) + Im(Z_p)$$

$$Z_{S_i} := R_{S_i} + j\cdot X_{S_i}$$

$$NF(Z_S) := NF_{min} + \sqrt{Re(Z_S)^2 + Im(Z_S)^2}$$

$$Gain(Z_S) := G_{max} + \sqrt{Re(Z_S)^2 + Im(Z_S)^2}$$



Other Equations and Calculations

$$F(\Gamma_S, F_{min}, R_N, \Gamma_{opt}) := F_{min} + \frac{4 \cdot R_N}{Z_0} \cdot \frac{(|\Gamma_S - \Gamma_{opt}|)^2}{\left[1 - (|\Gamma_S|)^2\right] \cdot (|1 + \Gamma_{opt}|)^2}$$

Noise Figure

$$G_T(\Gamma_S, \Gamma_L, S) := \frac{\left[1 - (|\Gamma_L|)^2\right] \cdot (|S_{2,1}|)^2 \cdot \left[1 - (|\Gamma_S|)^2\right]}{\left[\left|(1 - S_{2,2} \cdot \Gamma_L) \cdot (1 - S_{1,1} \cdot \Gamma_S) - S_{1,2} \cdot S_{2,1} \cdot \Gamma_L \cdot \Gamma_S\right|\right]^2}$$

Transducer Gain (Available Power Gain)

$$F_{sys}(\Gamma_S) := F(\Gamma_S, F_{min}, R_N, \Gamma_{opt}) + \frac{F_L - 1}{G_T(\Gamma_S, \Gamma_L, S)}$$

FL is the backend noise figure (mixer + etc.),which is not in dB.

$$\Gamma_S = \frac{Z_S - 50ohm}{Z_S + 50ohm} \qquad \Gamma_{in} = \frac{Z_{in} - 50ohm}{Z_{in} + 50ohm} \qquad S_{11} = \frac{Z_S - Z_{in}}{Z_S + Z_{in}}$$

$$Z_S = 50 \cdot ohm \cdot \frac{(\Gamma_S + 1)}{(1 - \Gamma_S)} \qquad Z_{in} = 50ohm \frac{(\Gamma_{in} + 1)}{(1 - \Gamma_{in})} \qquad S_{11}(\Gamma_S, \Gamma_{in}) := \frac{\Gamma_S - \Gamma_{in}}{1 - \Gamma_S \cdot \Gamma_{in}}$$

Copyright and Trademark Notice