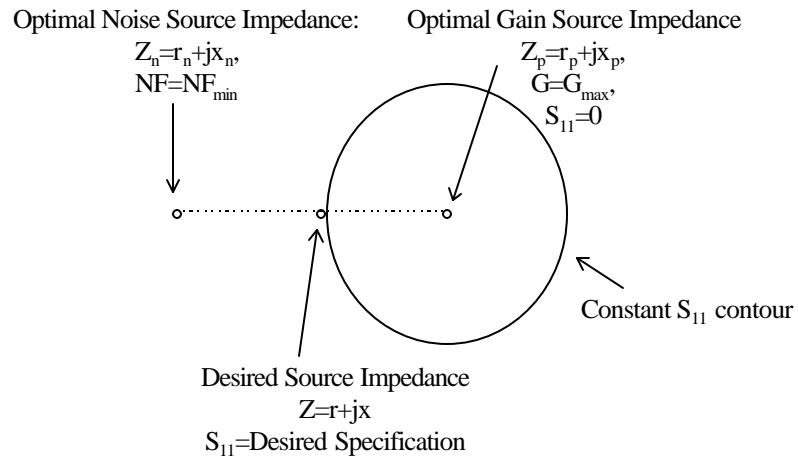


# Optimal Source Impedance to Trade-Off Power and Noise and to Achieve a Desired S11



- ▢ useful functions and identities
- ▢ Units
- ▢ Constants

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## Introduction

In the design of a low noise amplifier, LNA, or mixer driver stage there exists an optimal source impedance to provide the maximum power transfer into the transistor and thus the maximum gain. There also exists an optimal source impedance,  $Z_{nopt}$ , to provide the minimum noise figure for the circuit. At the maximum power source impedance,  $Z_{popt}$ , the input is matched to the source impedance and no reflections occur. This is desirable, because the filters attached to designed for the LNA or mixer, are designed for a matched load impedance, but can tolerate a certain amount of deviation from it's desired impedance. The amount of acceptable deviation is usually specified by the maximum return loss, or  $S_{11}$ , the filter can handle. The routine below finds the source impedance to the LNA or mixer, which will provide the lowest noise figure and maximum gain, while maintaining a desired  $S_{11}$  specification.

The calculation is performed by drawing a straight line between the optimal noise and power source impedances and finding where it intersects the desired  $S_{11}$  contour. This line intersects the contour at two points, so the point must be chosen, which is closest to the optimal noise source impedance. Sometimes, the desired  $S_{11}$  specification is achieved at the optimal noise source impedance. In this case the optimal noise source impedance is chosen over the intersecting point. This calculation is not optimal in the truest sense, because the noise and power gain circles are not concentric, but it serves as an excellent estimate to meet design criteria.

A few important notes: 1. The optimal source impedance for minimum noise figure should be calculated with estimates of ALL back-end noise components (mixers, PGA's, filters, A/Ds). Doing so will push the noise match much closer to the power match. 2. Simultaneous input-output match of a device (i.e optimum power match) may be unstable, which will push the desired source impedance closer to the noise match.

## Derivation

$$Z_S = r + j \cdot x \quad r = \text{Re}(Z_S) \quad x = \text{Im}(Z_S)$$

$$Z_n = r_n + j \cdot x_n \quad r_n = \text{Re}(Z_n) \quad x_n = \text{Im}(Z_n)$$

$$Z_p = r_p + j \cdot x_p \quad r_p = \text{Re}(Z_p) \quad x_p = \text{Im}(Z_p)$$

$$Z_{in} = \overline{Z_p} \quad r_i = \text{Re}(Z_{in}) \quad x_i = \text{Im}(Z_{in})$$

$$m = \frac{x_p - x_n}{r_p - r_n}$$

$$x = \frac{x_p - x_n}{r_p - r_n} \cdot (r - r_n) + x_n \quad x = m \cdot (r - r_n) + x_n$$

$$S_{11} = \left| \frac{Z_{in} - \overline{Z_S}}{Z_{in} + Z_S} \right| \quad S_{11} = 10^{\frac{S_{11dB}}{20}}$$

$$S_{11dB} = 20 \cdot \log \left( \left| \frac{r_i + j \cdot x_i - r + j \cdot x}{r_i + j \cdot x_i + r + j \cdot x} \right| \right)$$

$$S_{11}^2 \cdot (r_i + r)^2 = (r_i - r)^2 + (x_i + x)^2 \cdot (1 - S_{11}^2)$$

$$0 = r_i^2 - 2 \cdot r_i \cdot r \cdot \left( \frac{1 + S_{11}^2}{1 - S_{11}^2} \right) + r^2 + (x_i + x)^2$$

Equation Number 2

$$g = \frac{1 + S_{11}^2}{1 - S_{11}^2}$$

Inserting equation number of x into equation number two for x

$$0 = r_i^2 - 2 \cdot r_i \cdot r \cdot g + r^2 + [x_i + m \cdot (r - r_n) + x_n]^2$$

$$0 = (1 + m^2) \cdot r^2 + [-2 \cdot r_i \cdot g + 2 \cdot (x_i - m \cdot r_n + x_n) \cdot m] \cdot r + r_i^2 + (x_i - m \cdot r_n + x_n)^2$$

$$a = 1 + m^2 \quad b = 2 \cdot (x_i + x_n - m \cdot r_n) \cdot m - 2 \cdot r_i \cdot g \quad c = r_i^2 + (x_i + x_n - m \cdot r_n)^2$$

$$0 = a \cdot r^2 + b \cdot r + c$$

$$r = \left[ \frac{-1}{(2 \cdot a)} \cdot (b + \sqrt{b^2 - 4 \cdot a \cdot c}) \right] \quad x = m \cdot (r - r_n) + x_n$$

$$r = \left[ \frac{-1}{(2 \cdot a)} \cdot (b - \sqrt{b^2 - 4 \cdot a \cdot c}) \right]$$

Desired Source Impedance

Optimal Noise Source Impedance

Optimal Power Source Impedance

Input Impedance

Slope of line between Optimal Noise and Power Impedance

Equation Number 1: Straight line between noise and power impedances.

Input Match Equation

## Inputs

$$S_{11goal} := -10\text{dB}$$

$$Z_p := (21 + 10 \cdot j)\text{ohm}$$

$$Z_n := (10 + j \cdot 10)\text{ohm}$$

Desired  $S_{11}$  value

Optimal Source Impedance for Maximum Gain

Optimal Source Impedance for Minimum Noise Figure

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## Calculation

$$S_{11\text{goal}} := 10^{\frac{S_{11\text{goaldB}}}{20}} \quad S_{11\text{goal}} = 0.316$$

$$Z_{\text{in}} := \overline{Z_p} \quad Z_{\text{in}} = 21 - 10i \text{ ohm}$$

$$r_i := \text{Re}(Z_{\text{in}}) \quad x_i := \text{Im}(Z_{\text{in}}) \quad r_i = 21 \text{ ohm} \quad x_i = -10 \text{ ohm}$$

$$r_p := \text{Re}(Z_p) \quad x_p := \text{Im}(Z_p) \quad r_p = 21 \text{ ohm} \quad x_p = 10 \text{ ohm}$$

$$r_n := \text{Re}(Z_n) \quad x_n := \text{Im}(Z_n) \quad r_n = 10 \text{ ohm} \quad x_n = 10 \text{ ohm}$$

$$S_{11n} := 20 \cdot \log \left( \left| \frac{Z_{\text{in}} - \overline{Z_n}}{Z_{\text{in}} + Z_n} \right| \right) \quad S_{11n} = -8.999$$

$$g := \frac{1 + S_{11\text{goal}}^2}{1 - S_{11\text{goal}}^2} \quad g = 9.807 \text{ ms}^{-2}$$

$$m := \frac{x_p - x_n}{r_p - r_n} \quad m = 1 \text{ m}$$

$$a := 1 + m^2 \quad a = 1$$

$$b := 2 \cdot (x_i + x_n - m \cdot r_n) \cdot m - 2 \cdot r_i \cdot g \quad b = -51.333 \text{ ohm}$$

$$c := r_i^2 + (x_i + x_n - m \cdot r_n)^2 \quad c = 441 \text{ ohm}^2$$

$$r_1 := \frac{-1}{(2 \cdot a)} \cdot \left( b + \sqrt{b^2 - 4 \cdot a \cdot c} \right) \quad r_1 = 10.909 \text{ ohm}$$

$$x_1 := m \cdot (r_1 - r_n) + x_n \quad x_1 = 10 \text{ ohm}$$

$$d_1 := (r_1 - r_n)^2 + (x_1 - x_n)^2 \quad \sqrt{d_1} = 0.909 \text{ ohm}$$

$$r_2 := \frac{-1}{(2 \cdot a)} \cdot \left( b - \sqrt{b^2 - 4 \cdot a \cdot c} \right) \quad r_2 = 40.424 \text{ ohm}$$

$$x_2 := m \cdot (r_2 - r_n) + x_n \quad x_2 = 10 \text{ ohm}$$

$$d_2 := (r_2 - r_n)^2 + (x_2 - x_n)^2 \quad \sqrt{d_2} = 30.424 \text{ ohm}$$

$$r := \text{if}(d_1 < d_2, r_1, r_2) \quad r = 10.909 \text{ ohm}$$

$$x := \text{if}(d_1 < d_2, x_1, x_2) \quad x = 10 \text{ ohm}$$

$$Z_S := \text{if}(S_{11n} < S_{11\text{goaldB}}, Z_n, r + \sqrt{-1} \cdot x) \quad Z_S = 10.909 + 10i \text{ ohm}$$

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## Outputs

$$Z_S = 10.909 + 10i \text{ ohm}$$

### Check Results:

$$S_{11} := 20 \cdot \log \left( \left| \frac{Z_{\text{in}} - \overline{Z_S}}{Z_{\text{in}} + Z_S} \right| \right) \quad S_{11} = -10 \text{ dB}$$

## Function

$$\begin{aligned}
 Z_{Ss11}(Z_p, Z_n, S_{11goaldB}) := & \frac{S_{11goaldB}}{20} \\
 & S_{11goal} \leftarrow 10^{\frac{S_{11goaldB}}{20}} \\
 & Z_{in} \leftarrow \overline{Z_p} \\
 & r_i \leftarrow \operatorname{Re}(Z_{in}) \\
 & x_i \leftarrow \operatorname{Im}(Z_{in}) \\
 & r_p \leftarrow \operatorname{Re}(Z_p) \\
 & x_p \leftarrow \operatorname{Im}(Z_p) \\
 & r_n \leftarrow \operatorname{Re}(Z_n) \\
 & x_n \leftarrow \operatorname{Im}(Z_n) \\
 & S_{11n} \leftarrow 20 \cdot \log \left( \left| \frac{Z_{in} - \overline{Z_n}}{Z_{in} + Z_n} \right| \right) \\
 & g \leftarrow \frac{1 + S_{11goal}^2}{1 - S_{11goal}^2} \\
 & m \leftarrow \frac{x_p - x_n}{r_p - r_n} \\
 & a \leftarrow 1 + m^2 \\
 & b \leftarrow 2 \cdot (x_i + x_n - m \cdot r_n) \cdot m - 2 \cdot r_i \cdot g \\
 & c \leftarrow r_i^2 + (x_i + x_n - m \cdot r_n)^2 \\
 & r_1 \leftarrow \frac{-1}{(2 \cdot a)} \cdot (b + \sqrt{b^2 - 4 \cdot a \cdot c}) \\
 & x_1 \leftarrow m \cdot (r_1 - r_n) + x_n \\
 & d_1 \leftarrow (r_1 - r_n)^2 + (x_1 - x_n)^2 \\
 & r_2 \leftarrow \frac{-1}{(2 \cdot a)} \cdot (b - \sqrt{b^2 - 4 \cdot a \cdot c}) \\
 & x_2 \leftarrow m \cdot (r_2 - r_n) + x_n \\
 & d_2 \leftarrow (r_2 - r_n)^2 + (x_2 - x_n)^2 \\
 & r \leftarrow \operatorname{if}(d_1 < d_2, r_1, r_2) \\
 & x \leftarrow \operatorname{if}(d_1 < d_2, x_1, x_2) \\
 & Z_{Sans} \leftarrow \operatorname{if}(S_{11n} < S_{11goaldB}, Z_n, r + \sqrt{-1} \cdot x) \\
 & Z_{Sans}
 \end{aligned}$$

## Example

$$S_{11\text{goaldB}} := -15\text{dB}$$

$$Z_p := (50 + j \cdot 10)\text{ohm}$$

$$Z_n := (90 + j \cdot 10)\text{ohm}$$

$$G_{\text{max}} := 15\text{dB}$$

$$NF_{\text{min}} := 1.1\text{dB}$$

$$Z_{\text{in}} := \overline{Z_p}$$

$$Z_{\text{in}} = 50 - 10i\text{ ohm}$$

$$Z_S := Z_{Ss11}(Z_p, Z_n, S_{11\text{goaldB}})$$

$$Z_S = 71.629 + 10i\text{ ohm}$$

$$S_{11} := 20 \cdot \log \left( \left| \frac{Z_{\text{in}} - \overline{Z_S}}{Z_{\text{in}} + Z_S} \right| \right)$$

$$S_{11} = -15\text{ dB}$$

$$\text{if}(S_{11n} < S_{11\text{goaldB}}, Z_n, r + \sqrt{-1 \cdot x}) = 10.909 + 10i\text{ ohm}$$

## Plots

$$\text{num} := 100$$

$$i := 1.. \text{num}$$

$$R_{\text{Save}} := \frac{\text{Re}(Z_n) + \text{Re}(Z_p)}{2}$$

$$R_{\text{Sstart}} := \frac{R_{\text{Save}}}{5} \quad R_{\text{Sstop}} := R_{\text{Save}} \cdot 5$$

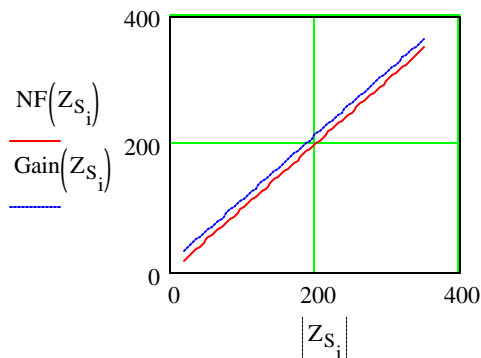
$$R_{S_i} := R_{\text{Sstart}} + \frac{i-1}{\text{num}-1} \cdot (R_{\text{Sstop}} - R_{\text{Sstart}})$$

$$X_{S_i} := \frac{\text{Im}(Z_n) - \text{Im}(Z_p)}{\text{Re}(Z_n) - \text{Re}(Z_p)} \cdot (R_{S_i} - \text{Re}(Z_p)) + \text{Im}(Z_p)$$

$$Z_{S_i} := R_{S_i} + j \cdot X_{S_i}$$

$$NF(Z_S) := NF_{\text{min}} + \sqrt{\text{Re}(Z_S)^2 + \text{Im}(Z_S)^2}$$

$$\text{Gain}(Z_S) := G_{\text{max}} + \sqrt{\text{Re}(Z_S)^2 + \text{Im}(Z_S)^2}$$



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## Other Equations and Calculations

$$F(\Gamma_S, F_{\min}, R_N, \Gamma_{\text{opt}}) := F_{\min} + \frac{4 \cdot R_N}{Z_0} \cdot \frac{(|\Gamma_S - \Gamma_{\text{opt}}|)^2}{\left[1 - (|\Gamma_S|)^2\right] \cdot (|1 + \Gamma_{\text{opt}}|)^2}$$

Noise Figure

$$G_T(\Gamma_S, \Gamma_L, S) := \frac{\left[1 - (|\Gamma_L|)^2\right] \cdot (|S_{2,1}|)^2 \cdot \left[1 - (|\Gamma_S|)^2\right]}{\left[\left|(1 - S_{2,2} \cdot \Gamma_L) \cdot (1 - S_{1,1} \cdot \Gamma_S) - S_{1,2} \cdot S_{2,1} \cdot \Gamma_L \cdot \Gamma_S\right|\right]^2}$$

Transducer Gain (Available Power Gain)

$$F_{\text{sys}}(\Gamma_S) := F(\Gamma_S, F_{\min}, R_N, \Gamma_{\text{opt}}) + \frac{F_L - 1}{G_T(\Gamma_S, \Gamma_L, S)}$$

FL is the backend noise figure (mixer + etc.), which is not in dB.

$$\Gamma_S = \frac{Z_S - 50\text{ohm}}{Z_S + 50\text{ohm}} \quad \Gamma_{\text{in}} = \frac{Z_{\text{in}} - 50\text{ohm}}{Z_{\text{in}} + 50\text{ohm}} \quad S_{11} = \frac{Z_S - Z_{\text{in}}}{Z_S + Z_{\text{in}}}$$

$$Z_S = 50\text{-ohm} \cdot \frac{(\Gamma_S + 1)}{(1 - \Gamma_S)} \quad Z_{\text{in}} = 50\text{ohm} \cdot \frac{(\Gamma_{\text{in}} + 1)}{(1 - \Gamma_{\text{in}})} \quad S_{11}(\Gamma_S, \Gamma_{\text{in}}) := \frac{\Gamma_S - \Gamma_{\text{in}}}{1 - \Gamma_S \cdot \Gamma_{\text{in}}}$$

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