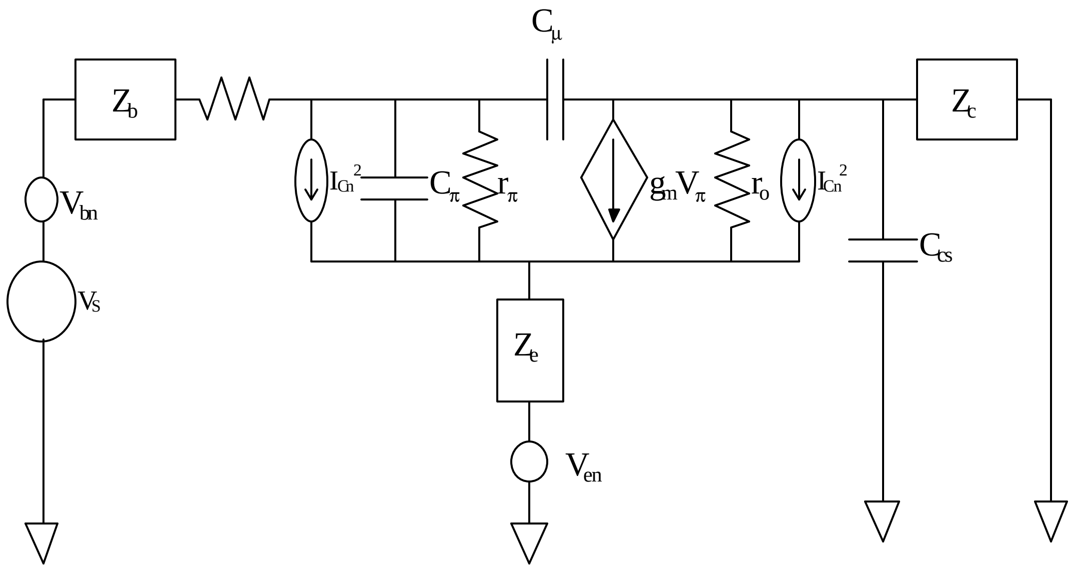
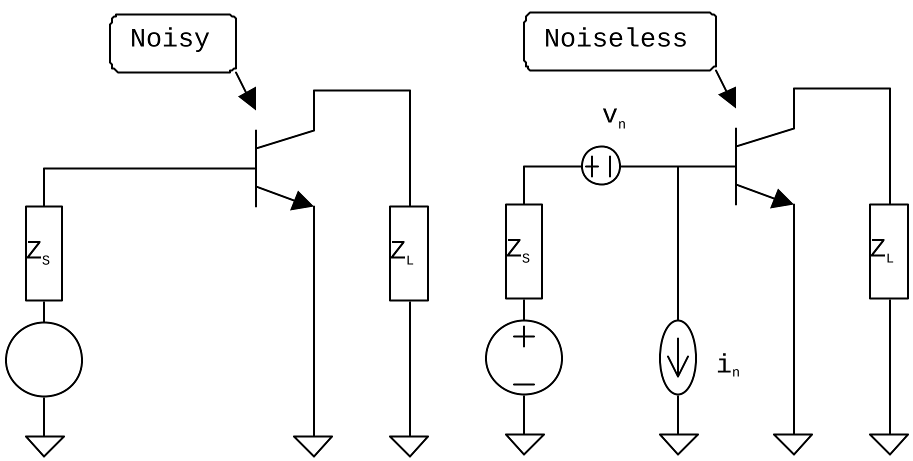
Low Noise Amplifier Optimization





A Few Notes on Notation

Note on Mathcad Notation: $\overline{\mathbf{X}}$ = X^*, the conjugate of X.

Note on Gains: $\mathbf{A_{r0}}$ = Gain from Base Resistance Noise to Output with Z_S=0.

$\mathbf{A_{b0}}$ = Gain from Base Current Noise to Output with Z_S=0.

$\mathbf{A_{c0}}$ = Gain from Collector Current Noise to Output with Z_S=0.

$\mathbf{A_{v}}$ = Gain from Input Voltage to Output with Z_S=0.

$\mathbf{A_{rinf}}$ = Gain from Base Resistance Noise to Output with Z_S=Infinity.

$\mathbf{A_{binf}}$ = Gain from Base Current Noise to Output with Z_S=Infinity.

$\mathbf{A_{cinf}}$ = Gain from Collector Current Noise to Output with Z_S=Inf.

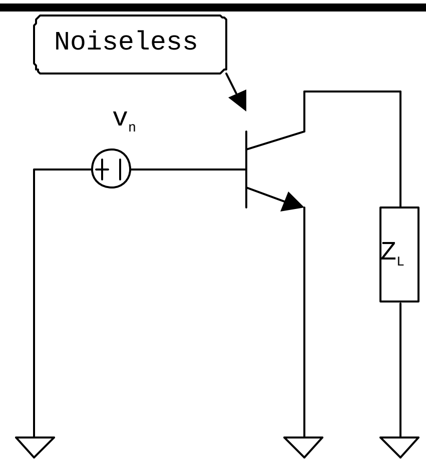
$\mathbf{A_{i}}$ = Gain from Input Current to Output with Z_S=Infinity.

$\mathbf{v_{nrb}}$ = Thermal Voltage Noise from Base Resistance

$\mathbf{I_{nB}}$ = Shot Current Noise from Base Current

$\mathbf{I_{nC}}$ = Shot Current Noise from Collector Current

Input-Referred Voltage Noise Source



$$Z_S = 0$$

$$V_{neq} = \frac{v_{nrb} \cdot A_{r0} + I_{nC} \cdot A_{c0} + I_{nB} \cdot A_{b0}}{A_v}$$

$$V_{neq} \cdot \overline{V_{neq}} = \begin{pmatrix} v_{nrb} \cdot \overline{v_{nrb}} \cdot A_{r0} \cdot \overline{A_{r0}} + v_{nrb} \cdot \overline{I_{nC}} \cdot A_{r0} \cdot \overline{A_{c0}} + v_{nrb} \cdot \overline{I_{nB}} \cdot A_{r0} \cdot \overline{A_{b0}} \ \dots \\ + I_{nC} \cdot \overline{v_{nrb}} \cdot A_{c0} \cdot \overline{A_{r0}} + I_{nC} \cdot \overline{I_{nC}} \cdot A_{c0} \cdot \overline{A_{c0}} + I_{nC} \cdot \overline{I_{nB}} \cdot A_{c0} \cdot \overline{A_{b0}} \ \dots \\ + I_{nB} \cdot \overline{v_{nrb}} \cdot A_{b0} \cdot \overline{A_{r0}} + I_{nB} \cdot \overline{I_{nC}} \cdot A_{b0} \cdot \overline{A_{c0}} + I_{nB} \cdot \overline{I_{nB}} \cdot A_{b0} \cdot \overline{A_{b0}} \end{pmatrix} \cdot \frac{1}{A_v \cdot \overline{A_v}}$$

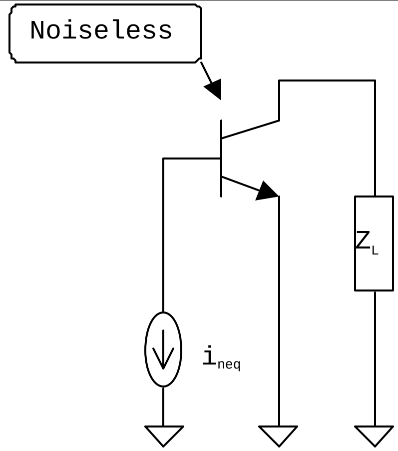
Take Expected Value: Cross Correlation Terms are Zero.

$$E\left[\left(\left|V_{neq}\right|\right)^2\right] = \frac{4 \cdot k \cdot T \cdot r_b \cdot \left(\left|A_{r0}\right|\right)^2 + 2 \cdot q \cdot I_C \cdot \left(\left|A_{c0}\right|\right)^2 + 2 \cdot q \cdot I_B \cdot \left(\left|A_{b0}\right|\right)^2}{\left(\left|A_v\right|\right)^2}$$

$$R_n = \frac{E\left[\left(\left|V_{neq}\right|\right)^2\right]}{4 \cdot k \cdot T}$$

Equivalent Noise Resistance

Input-Referred Current Noise Source



Z_S = ¥

$$I_{neq} = \frac{v_{nrb} \cdot A_{rinf} + I_{nC} \cdot A_{cinf} + I_{nB} \cdot A_{binf}}{A_i}$$

$$I_{neq} \cdot \overline{I_{neq}} = \begin{pmatrix} v_{nrb} \cdot \overline{v_{nrb}} \cdot A_{rinf} \cdot \overline{A_{rinf}} + v_{nrb} \cdot \overline{I_{nC}} \cdot A_{rinf} \cdot \overline{A_{cinf}} + v_{nrb} \cdot \overline{I_{nB}} \cdot A_{rinf} \cdot \overline{A_{binf}} \ \ldots \\ + I_{nC} \cdot \overline{v_{nrb}} \cdot A_{cinf} \cdot \overline{A_{rinf}} + I_{nC} \cdot \overline{I_{nC}} \cdot A_{cinf} \cdot \overline{A_{cinf}} + I_{nC} \cdot \overline{I_{nB}} \cdot A_{cinf} \cdot \overline{A_{binf}} \ \ldots \\ + I_{nB} \cdot \overline{v_{nrb}} \cdot A_{binf} \cdot \overline{A_{rinf}} + I_{nB} \cdot \overline{I_{nC}} \cdot A_{binf} \cdot \overline{A_{cinf}} + I_{nB} \cdot \overline{I_{nB}} \cdot A_{binf} \cdot \overline{A_{binf}} \end{pmatrix} \cdot \frac{1}{A_i \cdot \overline{A_i}}$$

Take Expected Value: Cross Correlation Terms are Zero.

$$E\left[\left(\left|I_{neq}\right|\right)^2\right] = \frac{4 \cdot k \cdot T \cdot r_b \cdot \left(\left|A_{rinf}\right|\right)^2 + 2 \cdot q \cdot I_C \cdot \left(\left|A_{cinf}\right|\right)^2 + 2 \cdot q \cdot I_B \cdot \left(\left|A_{binf}\right|\right)^2}{\left(\left|A_i\right|\right)^2}$$

$$g_n = \frac{E\left[\left(\left|I_{neq}\right|\right)^2\right]}{4 \cdot k \cdot T}$$ Equivalent Noise Conductance

Voltage and Current Noise Correlation

$$g = \frac{V_{neq} \cdot \overline{I_{neq}}}{\sqrt{V_{neq}^2 \cdot I_{neq}^2}}$$

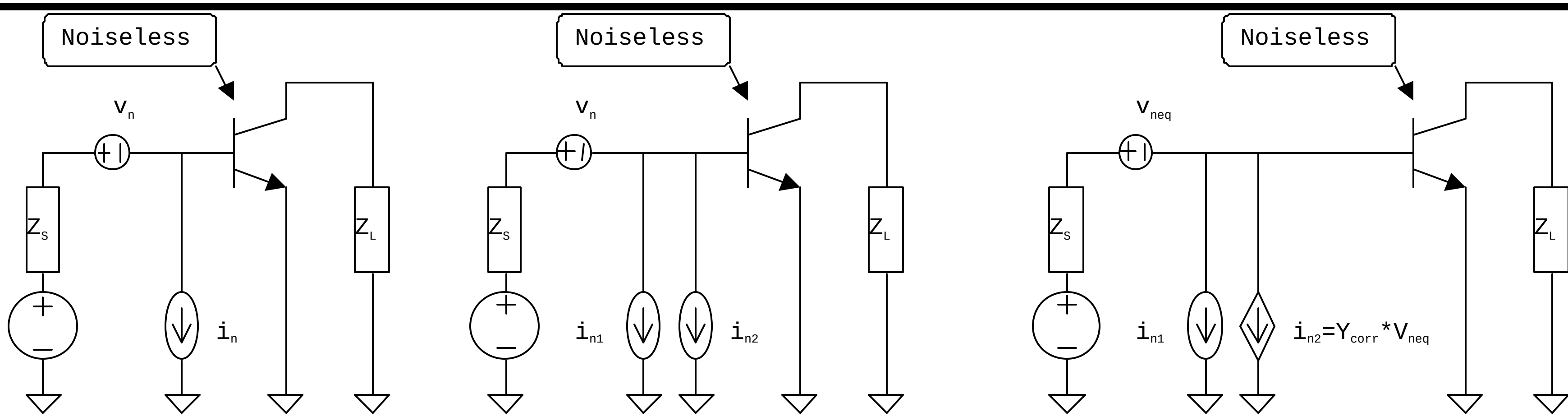
Correlation Coefficient:
|γ|=1: 100% correlated
|γ|=0: 0% correlelated

$$V_{neq} \cdot \overline{I_{neq}} = \begin{pmatrix} v_{nrb} \cdot \overline{v_{nrb}} \cdot A_{r0} \cdot \overline{A_{rinf}} + v_{nrb} \cdot \overline{I_{nC}} \cdot A_{r0} \cdot \overline{A_{cinf}} + v_{nrb} \cdot \overline{I_{nB}} \cdot A_{r0} \cdot \overline{A_{binf}} \; \ldots \\ + I_{nC} \cdot \overline{v_{nrb}} \cdot A_{c0} \cdot \overline{A_{rinf}} + I_{nC} \cdot \overline{I_{nC}} \cdot A_{c0} \cdot \overline{A_{cinf}} + I_{nC} \cdot \overline{I_{nB}} \cdot A_{c0} \cdot \overline{A_{binf}} \; \ldots \\ + I_{nB} \cdot \overline{v_{nrb}} \cdot A_{b0} \cdot \overline{A_{rinf}} + I_{nB} \cdot \overline{I_{nC}} \cdot A_{b0} \cdot \overline{A_{cinf}} + I_{nB} \cdot \overline{I_{nB}} \cdot A_{b0} \cdot \overline{A_{binf}} \end{pmatrix} \cdot \frac{1}{A_i \cdot \overline{A_i}}$$

Take Expected Value: Cross Correlation Terms are Zero.

$$E\left(V_{neq} \cdot \overline{I_{neq}}\right) = \frac{4 \cdot k \cdot T \cdot r_b \cdot A_{r0} \cdot \overline{A_{rinf}} + 2 \cdot q \cdot I_C \cdot A_{c0} \cdot \overline{A_{cinf}} + 2 \cdot q \cdot I_B \cdot A_{b0} \cdot \overline{A_{binf}}}{A_v \cdot \overline{A_i}}$$

Correlation Admittance



How do we find Y_{corr}?

$$E\left(i_{n1}\cdot V_{neq}\right) = 0$$

$$i_{n2} = Y_{corr}\cdot V_{neq}$$

$$i_{n2}\cdot\overline{V_{neq}} = Y_{corr}\cdot V_{neq}\cdot\overline{V_{neq}}$$

$$E\left[\left(i_n - i_{n1}\right)\cdot\overline{V_{neq}}\right] = Y_{corr}\cdot E\left(V_{neq}\cdot\overline{V_{neq}}\right)$$

$$Y_{corr} = \frac{E\left(i_n\cdot\overline{V_{neq}}\right) - E\left(i_{n1}\cdot\overline{V_{neq}}\right)}{E\left(V_{neq}\cdot\overline{V_{neq}}\right)} = \frac{\overline{E\left(V_{neq}\cdot\overline{I_{neq}}\right)}}{V_{neq}{}^2}$$

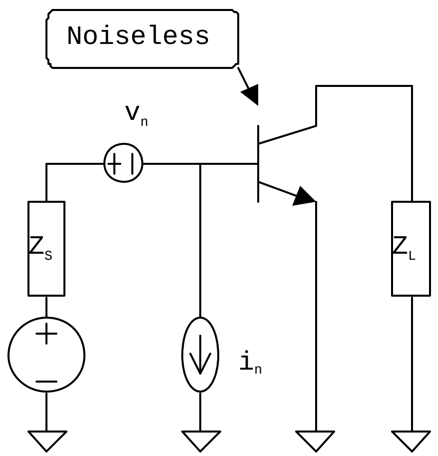
Substitute the following:

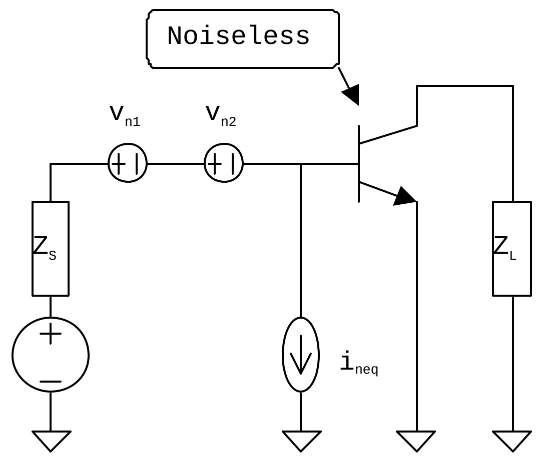
$$E\left(V_{neq}\cdot\overline{I_{neq}}\right) = g\cdot\sqrt{V_{neq}{}^2\cdot I_{neq}{}^2}$$

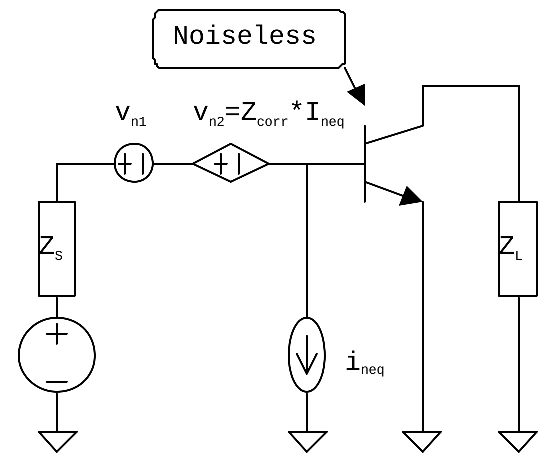
$$Y_{corr} = \overline{g}\cdot\sqrt{\frac{E\left[\left(\left|I_{neq}\right|\right)^2\right]}{E\left[\left(\left|V_{neq}\right|\right)^2\right]}}$$

$$G_{corr} = \mathrm{Re}\left(Y_{corr}\right) \qquad B_{corr} = \mathrm{Im}\left(Y_{corr}\right)$$

Correlation Impedance







How do we find Z_{corr}?

$$E\left(V_{n1}\cdot I_{neq}\right) = 0$$

$$V_{n2} = Z_{corr}\cdot I_{neq}$$

$$V_{n2}\cdot\overline{I_{neq}} = Z_{corr}\cdot I_{neq}\cdot\overline{I_{neq}}$$

$$E\left[\left(V_{neq} - V_{n1}\right)\cdot\overline{I_{neq}}\right] = Z_{corr}\cdot E\left(I_{neq}\cdot\overline{I_{neq}}\right)$$

$$Z_{corr} = \frac{E\left(V_{neq}\cdot\overline{i_{neq}}\right) - E\left(i_{n1}\cdot\overline{V_{neq}}\right)}{E\left(I_{neq}\cdot\overline{I_{neq}}\right)} = \frac{E\left(V_{neq}\cdot\overline{I_{neq}}\right)}{I_{neq}^{\;2}}$$

Substitute the following:

$$E\left(V_{neq}\cdot\overline{I_{neq}}\right) = g\cdot\sqrt{V_{neq}^{\;2}\cdot I_{neq}^{\;2}}$$

$$Z_{corr} = g\cdot\sqrt{\frac{E\left[\left(\left|V_{neq}\right|\right)^2\right]}{E\left[\left(\left|I_{neq}\right|\right)^2\right]}}$$

$$R_{corr} = \mathrm{Re}\left(Z_{corr}\right) \qquad X_{corr} = \mathrm{Im}\left(Z_{corr}\right)$$

Uncorrelated Noise Voltage

r_n is the noise resistance representing the portion of V_{neq}^2 which is uncorrelated with I_{neq}^2. It is always less than R_n, and in the case where the noise is 100% correlated, it is equal to 0.

$$\mathbf{V_{neq}} \cdot \overline{\mathbf{V_{neq}}} = (\mathbf{V_{n1}} + \mathbf{V_{n2}}) \cdot (\overline{\mathbf{V_{n1}}} + \overline{\mathbf{V_{n2}}})$$

$$\mathbf{E}\left[(|\mathbf{V_{neq}}|)^2\right] = \mathbf{E}(\mathbf{V_{n1}} \cdot \overline{\mathbf{V_{n1}}}) + \mathbf{E}(\mathbf{V_{n1}} \cdot \overline{\mathbf{V_{n2}}}) + \mathbf{E}(\mathbf{V_{n2}} \cdot \overline{\mathbf{V_{n1}}}) + \mathbf{E}(\mathbf{V_{n2}} \cdot \overline{\mathbf{V_{n2}}})$$

$$\mathbf{E}\left[(|\mathbf{v_{n1}}|)^2\right] = \mathbf{E}\left(\left|\mathbf{V_{neq}}^2\right|\right) - (|\mathbf{Z_{corr}}|)^2 \cdot \mathbf{E}\left(\left|\mathbf{I_{neq}}^2\right|\right)$$

$$\mathbf{r_n} = \frac{\mathbf{E}\left[(|\mathbf{v_{n1}}|)^2\right]}{4 \cdot \mathbf{k} \cdot \mathbf{T}} = \mathbf{R_n} - \mathbf{g_n} \cdot (|\mathbf{Z_{corr}}|)^2 \qquad \mathbf{r_n} = \mathbf{R_n} \cdot \left[1 - (|g|)^2\right]$$

Note: R_n<>r_n!

Uncorrelated Noise Current

G_n is the noise conductance representing the portion of I_{neq}^2 which is uncorrelated with V_{neq}^2. It is always less than g_n, and in the case where the noise is 100% correlated, it is equal to 0.

$$\mathbf{I_{neq}} \cdot \overline{\mathbf{I_{neq}}} = (\mathbf{i_{n1}} + \mathbf{i_{n2}}) \cdot \left(\overline{\mathbf{i_{n1}}} + \overline{\mathbf{i_{n2}}}\right)$$

$$\mathbf{E}\left[\left(\left|\mathbf{I_{neq}}\right|\right)^2\right] = \mathbf{E}\left(\mathbf{i_{n1}} \cdot \overline{\mathbf{i_{n1}}}\right) + \mathbf{E}\left(\mathbf{i_{n1}} \cdot \overline{\mathbf{i_{n2}}}\right) + \mathbf{E}\left(\mathbf{i_{n2}} \cdot \overline{\mathbf{i_{n1}}}\right) + \mathbf{E}\left(\mathbf{i_{n2}} \cdot \overline{\mathbf{i_{n2}}}\right)$$

$$\mathbf{E}\left[\left(\left|\mathbf{i_{n1}}\right|\right)^2\right] = \mathbf{E}\left[\left(\left|\mathbf{I_{neq}}\right|\right)^2\right] - \left(\left|\mathbf{Y_{corr}}\right|\right)^2 \cdot \mathbf{E}\left[\left(\left|\mathbf{V_{neq}}\right|\right)^2\right]$$

$$\mathbf{G_n} = \frac{\mathbf{E}\left[\left(\left|\mathbf{i_{n1}}\right|\right)^2\right]}{4 \cdot \mathbf{k} \cdot \mathbf{T}} = \mathbf{g_n} - \mathbf{R_n} \cdot \left(\left|\mathbf{Z_{corr}}\right|\right)^2$$

$$\mathbf{G_n} = \mathbf{g_n} \cdot \left[1 - \left(\left|g\right|\right)^2\right]$$

Note: G_n<>g_n!

Noise Figure

Output-referred noise voltage with arbitrary source impedance Z_S

$$V_o = (V_{n1} + V_{n2} + V_{sn}) \cdot \frac{Z_{in}}{Z_{in} + Z_S} \cdot A + i_n \cdot \frac{Z_{in} \cdot Z_S}{Z_{in} + Z_S} \cdot A$$

Input-referred noise voltage with arbitrary source impedance Z_S.

$$V_{seq} = V_{n1} + V_{n2} + V_{sn} + I_{neq} \cdot Z_S = V_{n1} + I_{neq} \cdot Z_{corr} + V_{sn} + I_{neq} \cdot Z_S$$

$$V_{seq} \cdot \overline{V_{seq}} = \left[V_{n1} + V_{sn} + I_{neq} \cdot (Z_S + Z_{corr}) \right] \cdot \left[\overline{V_{n1}} + \overline{V_{sn}} + \overline{I_{neq}} \cdot \left(\overline{Z_S} + \overline{Z_{corr}} \right) \right]$$

Input-referred noise power (normalized to 1 ohm)

$$E\left[\left(\left| V_{seq} \right| \right)^2 \right] = E\left[\left(\left| V_{n1} \right| \right)^2 \right] + E\left[\left(\left| V_{sn} \right| \right)^2 \right] + E\left[\left(\left| I_{neq} \right| \right)^2 \right] \cdot \left(\left| Z_S + Z_{corr} \right| \right)^2$$

Noise figure = (Input-referred noise power)/(Source noise power)

$$F = \frac{E\left[\left(\left| V_{seq} \right| \right)^2 \right]}{E\left[\left(\left| V_{sn} \right| \right)^2 \right]} = 1 + \frac{{V_{n1}}^2}{{V_{sn}}^2} + \frac{{I_{neq}}^2}{{V_{sn}}^2} \cdot \left(\left| Z_S + Z_{corr} \right| \right)^2$$

Substitute the following: ${V_{sn}}^2 = 4 \cdot k \cdot T \cdot R_S$ ${I_{neq}}^2 = 4 \cdot k \cdot T \cdot g_n$ ${V_{n1}}^2 = 4 \cdot k \cdot T \cdot r_n$

$$F = 1 + \frac{r_n}{R_S} + \frac{g_n}{R_S} \cdot \left(\left| Z_S + Z_{corr} \right| \right)^2$$

Optimal Noise Source Impedance

Set Derivatives of F with Respect to X_S and R_S equal to zero and solve for X_S and R_S.

$$F = 1 + \frac{r_n}{R_S} + \frac{g_n}{R_S}\cdot\left[\left(R_S + R_{corr}\right)^2 + \left(X_S + X_{corr}\right)^2\right]$$

$$\frac{d}{dX_S}F = 0 = X_{Sopt} + X_{corr} \qquad X_{Sopt} = -X_{corr}$$

$$\frac{d}{dR_S}F = \frac{-r_n}{R_{Sopt}^2} + g_n\cdot\left(1 - \frac{R_{corr}^2}{R_{Sopt}^2}\right) = 0 \qquad R_{Sopt} = \sqrt{\frac{r_n}{g_n} + R_{corr}^2}$$

$$Z_{nopt} = \sqrt{\frac{r_n}{g_n} + R_{corr}^2} - j\cdot X_{corr}$$

Optimal Noise Figure

Plug Optimal Noise Source Impedance, Z_{nopt}, into Noise Figure Equation and Solve for F_{min}.

$$F_{min} = 1 + \frac{r_n}{R_S} + \frac{g_n}{R_S} \cdot \left[\left(\sqrt{\frac{r_n}{g_n} + R_{corr}^2} + R_{corr} \right)^2 + \left(-X_{corr} + X_{corr} \right)^2 \right]$$

$$F_{min} = 1 + 2 \cdot \left[g_n \cdot R_{corr} + \sqrt{g_n \cdot r_n + \left(g_n \cdot R_{corr} \right)^2} \right]$$

Example (Simplified BJT)

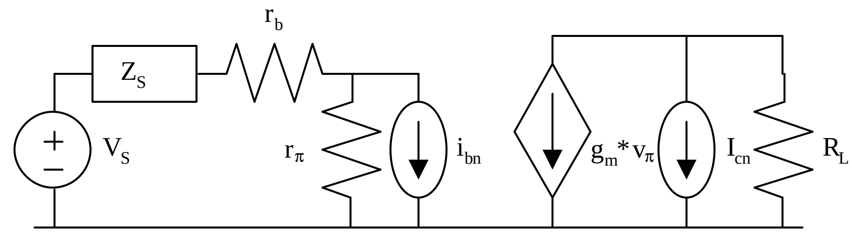
$r_b := 10\text{W}$ Base Resistance

$\text{b} := 100$ DC Beta

$I_C := 1\text{mA}$ Collector Current

$R_L := 1\text{kW}$ Load Resistance

$T := 300\text{K}$ Temperature



Constants

$k := 1.3806 \cdot 10^{-23} \cdot \frac{J}{K}$ Boltzman's Constant

$q := 1.602 \cdot 10^{-19} C$ Charge on an Electron

Small Signal Parameters

$V_T := \frac{k \cdot T}{q}$ $\quad V_T = 25.854\,\text{mV}$ Thermal Voltage

$g_m := \frac{I_C}{V_T}$ $\quad g_m = 38.679\,\frac{\text{mA}}{\text{V}}$ Device Transconductance

$I_B := \frac{I_C}{|\text{b}|}$ $\quad I_B = 10\,\text{mA}$ Base Current

$r_p := \frac{\text{b}}{g_m}$ $\quad r_p = 2.585\,\text{kW}$ Base Emitter Resistance

Gain Calculations for Example

$A_{r0} := \frac{r_p}{r_p + r_b} \cdot g_m \cdot R_L$ $A_{r0} = 38.53$ Base Resistance Gain with Z_S=0

$A_{b0} := \frac{r_b \cdot r_p}{r_p + r_b} \cdot g_m \cdot R_L$ $A_{b0} = 385.298\,\mathrm{ohm}$ Base Current Gain with Z_S=0

$A_{c0} := R_L$ $A_{c0} = 1\,\mathrm{kW}$ Collector Current Gain with Z_S=0

$A_v := \frac{r_p}{r_p + r_b} \cdot g_m \cdot R_L$ $A_v = 38.53$ Input Voltage Gain with Z_S=0

$A_{rinf} := 0$ $A_{rinf} = 0$ Base Resistance Gain with Z_S=inf.

$A_{binf} := r_p \cdot g_m \cdot R_L$ $A_{binf} = 100\,\mathrm{kW}$ Base Current Gain with Z_S=inf.

$A_{cinf} := R_L$ $A_{cinf} = 1\,\mathrm{kW}$ Collector Current Gain with Z_S=inf.

$A_i := r_p \cdot g_m \cdot R_L$ $A_i = 100\,\mathrm{kW}$ Input Current Gain with Z_S=inf.

Input Noise Calculations

Equivalent Input Noise Voltage

$$V_{neq} := \sqrt{4 \cdot k \cdot T \cdot r_b + 2 \cdot q \cdot I_B \cdot r_b{}^2 + 2 \cdot q \cdot I_C \cdot \left(\frac{1}{g_m}\right)^2 \cdot \left(\left|\frac{r_p + r_b}{r_p}\right|\right)^2}$$

$$V_{neq} = 0.618 \, \frac{nV}{\sqrt{Hz}}$$

Equivalent Input Noise Resistance

$$R_n := \frac{V_{neq}{}^2}{4 \cdot k \cdot T}$$

$$R_n = 23.046 \, \mathrm{W}$$

Equivalent Input Noise Current

$$I_{neq} := \sqrt{2 \cdot q \cdot I_B + \frac{2 \cdot q \cdot I_C}{\left(\left|g_m \cdot r_p\right|\right)^2}}$$

$$I_{neq} = 1.799 \, \frac{pA}{\sqrt{Hz}}$$

Equivalent Input Noise Conductance

$$g_n := \frac{I_{neq}{}^2}{4 \cdot k \cdot T}$$

$$\frac{1}{g_n} = 5.12 \, \mathrm{kW}$$

Noise Correlation

$$VI_{conj} := \frac{2 \cdot q \cdot I_C \cdot (r_p + r_b)}{\left(\left|g_m \cdot r_p\right|\right)^2} + 2 \cdot q \cdot I_B \cdot r_b$$

$$VI_{conj} = 1.152 \times 10^{-22} \frac{W}{Hz}$$

$$g := \frac{VI_{conj}}{\sqrt{V_{neq}{}^2 \cdot I_{neq}{}^2}}$$

$$g = 0.104$$

Correlation Coefficient

$$g_{guess} := \frac{1}{\sqrt{b}}$$

$$g_{guess} = 0.1$$

Correlation Guess (g_m * r_b <<1, β>>1)

$$Y_{corr} := \bar{g} \cdot \sqrt{\frac{I_{neq}{}^2}{V_{neq}{}^2}}$$

$$\frac{1}{Y_{corr}} = 3.314\,\mathrm{kW}$$

Correlation Conductance

$$Z_{corr} := g \cdot \sqrt{\frac{V_{neq}{}^2}{I_{neq}{}^2}}$$

$$Z_{corr} = 35.598\,\mathrm{W}$$

Correlation Impedance

$$r_n := R_n \cdot \left[1 - \left(\left|g\right|\right)^2\right]$$

$$r_n = 22.799\,\mathrm{W}$$

Correlation Resistance

$$G_n := g_n \cdot \left[1 - \left(\left|g\right|\right)^2\right]$$

$$\frac{1}{G_n} = 5.175\,\mathrm{kW}$$

Correlation Conductance

Optimal Noise Figure and Source Impedance

Optimal Source Impedance for Minimal Noise Figure

$$Z_{nopt} := \sqrt{\frac{r_n}{g_n} + \mathrm{Re}\left(Z_{corr}\right)^2} - j \cdot \mathrm{Im}\left(Z_{corr}\right) \qquad Z_{nopt} = 343.495\,\mathrm{ohm}$$

Optimal Noise Figure

$$NF_{min} := 10 \cdot \log\left[1 + 2 \cdot \left[g_n \cdot \mathrm{Re}\left(Z_{corr}\right) + \sqrt{g_n \cdot r_n + \left(g_n \cdot \mathrm{Re}\left(Z_{corr}\right)\right)^2}\right]\right] \qquad NF_{min} = 0.6\,\mathrm{dB}$$

Noise Figure Plots

$$NF(Z_S) := 10\cdot\log\left[1 + \frac{r_n}{Re(Z_S)} + \frac{g_n}{Re(Z_S)}\cdot\left(\left|Z_S + Z_{corr}\right|\right)^2\right]$$

$i := 1 .. 20$

$R_{S_i} := \frac{i}{20}\cdot 600ohm$ $\quad Z_{S_i} := R_{S_i} + j\cdot Im(Z_{nopt})$ $\quad$ Sweeping R_S

$X_{S_i} := \frac{i}{20}\cdot 400ohm - 200ohm$ $\quad Z_{S2_i} := Re(Z_{nopt}) + j\cdot X_{S_i}$ $\quad$ Sweeping X_S