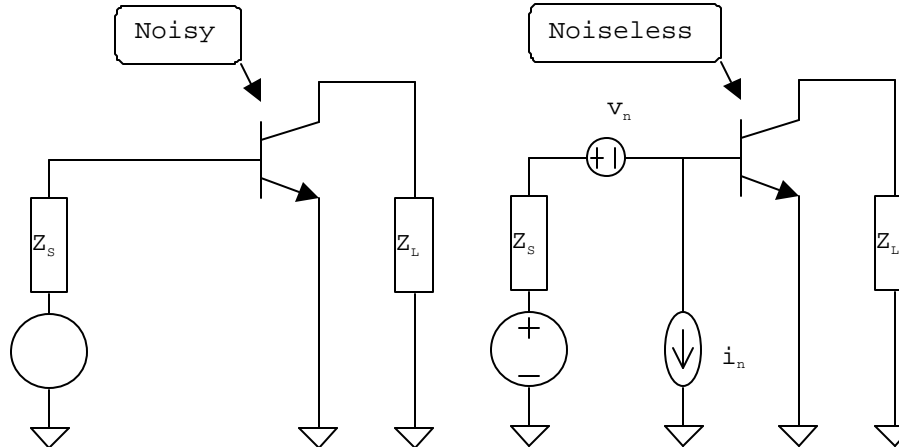
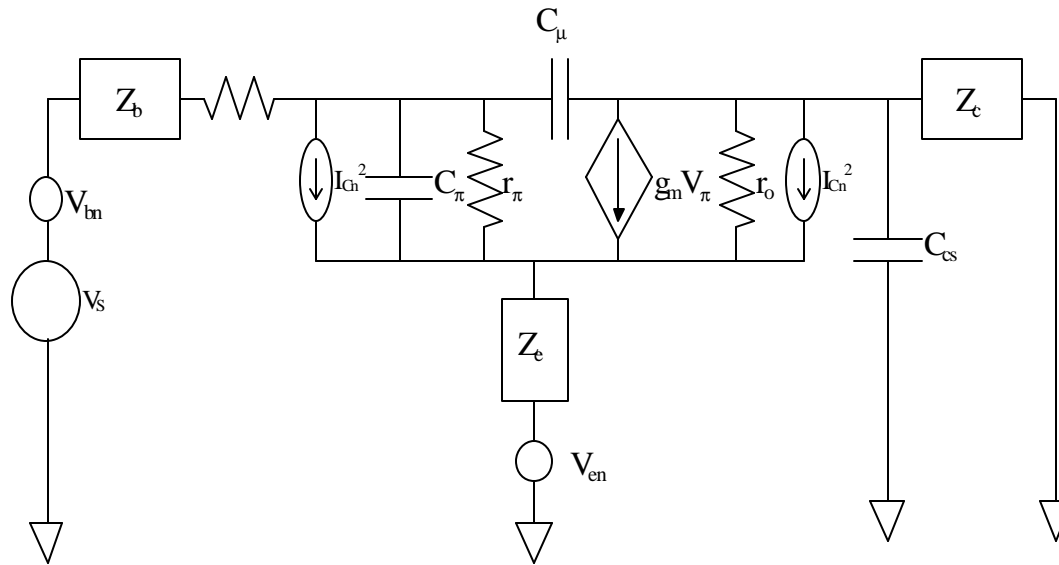


Low Noise Amplifier Optimization



A Few Notes on Notation

Note on Mathcad Notation: $\overline{\mathbf{X}} = \mathbf{X}^*$, the conjugate of \mathbf{X} .

Note on Gains: \mathbf{A}_{r0} = Gain from Base Resistance Noise to Output with $Z_S=0$.

\mathbf{A}_{b0} = Gain from Base Current Noise to Output with $Z_S=0$.

\mathbf{A}_{c0} = Gain from Collector Current Noise to Output with $Z_S=0$.

\mathbf{A}_v = Gain from Input Voltage to Output with $Z_S=0$.

\mathbf{A}_{rinf} = Gain from Base Resistance Noise to Output with $Z_S=I$ nfinity.

\mathbf{A}_{binf} = Gain from Base Current Noise to Output with $Z_S=I$ nfinity.

\mathbf{A}_{cinf} = Gain from Collector Current Noise to Output with $Z_S=I$ nf.

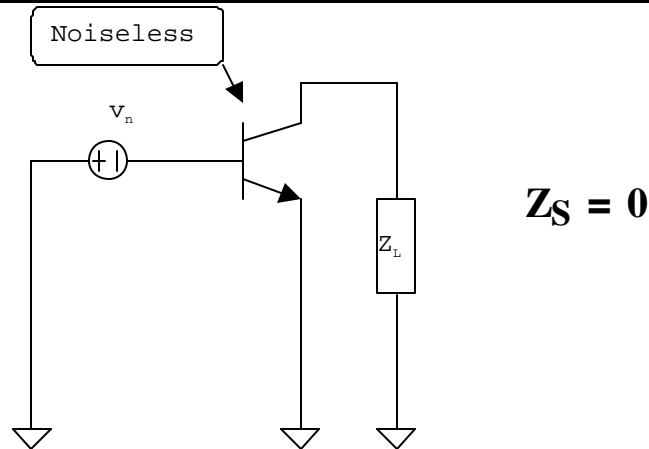
\mathbf{A}_i = Gain from Input Current to Output with $Z_S=I$ nfinity.

\mathbf{v}_{nrB} = Thermal Voltage Noise from Base Resistance

\mathbf{I}_{nB} = Shot Current Noise from Base Current

\mathbf{I}_{nC} = Shot Current Noise from Collector Current

Input-Referred Voltage Noise Source



$$V_{\text{neq}} = \frac{v_{\text{nrB}} \cdot A_{\text{r0}} + I_{\text{nC}} \cdot A_{\text{c0}} + I_{\text{nB}} \cdot A_{\text{b0}}}{A_{\text{v}}}$$

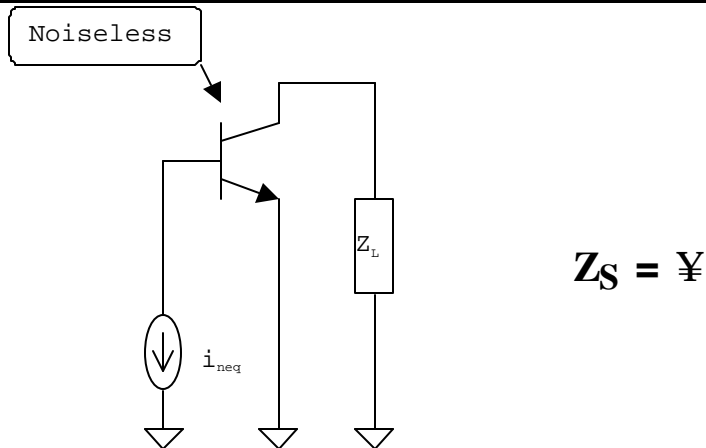
$$V_{\text{neq}} \cdot \overline{V_{\text{neq}}} = \left(\begin{array}{l} v_{\text{nrB}} \cdot \overline{v_{\text{nrB}}} \cdot A_{\text{r0}} \cdot \overline{A_{\text{r0}}} + v_{\text{nrB}} \cdot \overline{I_{\text{nC}}} \cdot A_{\text{r0}} \cdot \overline{A_{\text{c0}}} + v_{\text{nrB}} \cdot \overline{I_{\text{nB}}} \cdot A_{\text{r0}} \cdot \overline{A_{\text{b0}}} \dots \\ + I_{\text{nC}} \cdot \overline{v_{\text{nrB}}} \cdot A_{\text{c0}} \cdot \overline{A_{\text{r0}}} + I_{\text{nC}} \cdot \overline{I_{\text{nC}}} \cdot A_{\text{c0}} \cdot \overline{A_{\text{c0}}} + I_{\text{nC}} \cdot \overline{I_{\text{nB}}} \cdot A_{\text{c0}} \cdot \overline{A_{\text{b0}}} \dots \\ + I_{\text{nB}} \cdot \overline{v_{\text{nrB}}} \cdot A_{\text{b0}} \cdot \overline{A_{\text{r0}}} + I_{\text{nB}} \cdot \overline{I_{\text{nC}}} \cdot A_{\text{b0}} \cdot \overline{A_{\text{c0}}} + I_{\text{nB}} \cdot \overline{I_{\text{nB}}} \cdot A_{\text{b0}} \cdot \overline{A_{\text{b0}}} \end{array} \right) \cdot \frac{1}{A_{\text{v}} \cdot \overline{A_{\text{v}}}}$$

Take Expected Value: Cross Correlation Terms are Zero.

$$E\left[(|V_{\text{neq}}|)^2 \right] = \frac{4 \cdot k \cdot T \cdot r_{\text{b}} \cdot (|A_{\text{r0}}|)^2 + 2 \cdot q \cdot I_{\text{C}} \cdot (|A_{\text{c0}}|)^2 + 2 \cdot q \cdot I_{\text{B}} \cdot (|A_{\text{b0}}|)^2}{(|A_{\text{v}}|)^2}$$

$$R_{\text{n}} = \frac{E\left[(|V_{\text{neq}}|)^2 \right]}{4 \cdot k \cdot T} \quad \text{Equivalent Noise Resistance}$$

Input-Referred Current Noise Source



$$I_{neq} = \frac{v_{nrB} \cdot A_{rinf} + I_{nC} \cdot A_{cinf} + I_{nB} \cdot A_{binf}}{A_i}$$

$$I_{neq} \cdot \overline{I_{neq}} = \left(\begin{array}{l} v_{nrB} \cdot \overline{v_{nrB}} \cdot \overline{A_{rinf}} \cdot \overline{A_{rinf}} + v_{nrB} \cdot \overline{I_{nC}} \cdot \overline{A_{rinf}} \cdot \overline{A_{cinf}} + v_{nrB} \cdot \overline{I_{nB}} \cdot \overline{A_{rinf}} \cdot \overline{A_{binf}} \dots \\ + I_{nC} \cdot \overline{v_{nrB}} \cdot \overline{A_{cinf}} \cdot \overline{A_{rinf}} + I_{nC} \cdot \overline{I_{nC}} \cdot \overline{A_{cinf}} \cdot \overline{A_{cinf}} + I_{nC} \cdot \overline{I_{nB}} \cdot \overline{A_{cinf}} \cdot \overline{A_{binf}} \dots \\ + I_{nB} \cdot \overline{v_{nrB}} \cdot \overline{A_{binf}} \cdot \overline{A_{rinf}} + I_{nB} \cdot \overline{I_{nC}} \cdot \overline{A_{binf}} \cdot \overline{A_{cinf}} + I_{nB} \cdot \overline{I_{nB}} \cdot \overline{A_{binf}} \cdot \overline{A_{binf}} \dots \end{array} \right) \cdot \frac{1}{A_i \cdot A_i}$$

Take Expected Value: Cross Correlation Terms are Zero.

$$E\left[(|I_{neq}|)^2 \right] = \frac{4 \cdot k \cdot T \cdot r_b \cdot (|A_{rinf}|)^2 + 2 \cdot q \cdot I_C \cdot (|A_{cinf}|)^2 + 2 \cdot q \cdot I_B \cdot (|A_{binf}|)^2}{(|A_i|)^2}$$

$$g_n = \frac{E\left[(|I_{neq}|)^2 \right]}{4 \cdot k \cdot T} \quad \text{Equivalent Noise Conductance}$$

Voltage and Current Noise Correlation

$$\rho = \frac{\overline{V_{\text{neq}} \cdot I_{\text{neq}}}}{\sqrt{\overline{V_{\text{neq}}^2} \cdot \overline{I_{\text{neq}}^2}}}$$

Correlation Coefficient:

$|\rho|=1$: 100% correlated

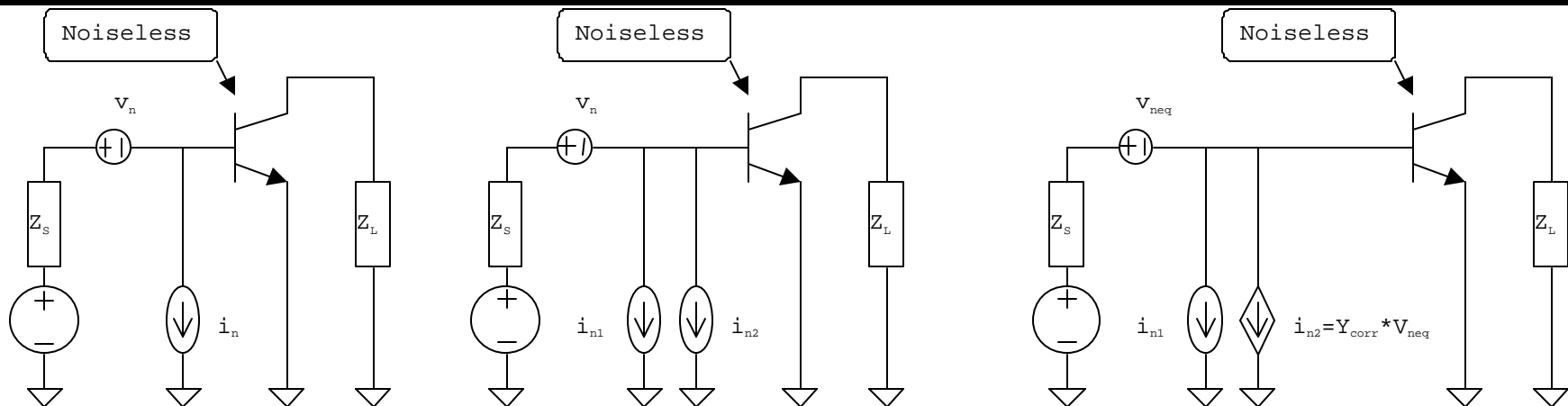
$|\rho|=0$: 0% correlated

$$\overline{V_{\text{neq}} \cdot I_{\text{neq}}} = \left(\begin{array}{l} \overline{v_{\text{nrB}} \cdot v_{\text{nrB}} \cdot A_{\text{r0}} \cdot A_{\text{rinf}}} + \overline{v_{\text{nrB}} \cdot I_{\text{nC}} \cdot A_{\text{r0}} \cdot A_{\text{cinf}}} + \overline{v_{\text{nrB}} \cdot I_{\text{nB}} \cdot A_{\text{r0}} \cdot A_{\text{binf}}} \dots \\ + \overline{I_{\text{nC}} \cdot v_{\text{nrB}} \cdot A_{\text{c0}} \cdot A_{\text{rinf}}} + \overline{I_{\text{nC}} \cdot I_{\text{nC}} \cdot A_{\text{c0}} \cdot A_{\text{cinf}}} + \overline{I_{\text{nC}} \cdot I_{\text{nB}} \cdot A_{\text{c0}} \cdot A_{\text{binf}}} \dots \\ + \overline{I_{\text{nB}} \cdot v_{\text{nrB}} \cdot A_{\text{b0}} \cdot A_{\text{rinf}}} + \overline{I_{\text{nB}} \cdot I_{\text{nC}} \cdot A_{\text{b0}} \cdot A_{\text{cinf}}} + \overline{I_{\text{nB}} \cdot I_{\text{nB}} \cdot A_{\text{b0}} \cdot A_{\text{binf}}} \end{array} \right) \cdot \frac{1}{A_{\text{i}} \cdot A_{\text{i}}}$$

Take Expected Value: Cross Correlation Terms are Zero.

$$\mathbf{E}(\overline{V_{\text{neq}} \cdot I_{\text{neq}}}) = \frac{4 \cdot k \cdot T \cdot r_{\text{b}} \cdot A_{\text{r0}} \cdot A_{\text{rinf}} + 2 \cdot q \cdot I_{\text{C}} \cdot A_{\text{c0}} \cdot A_{\text{cinf}} + 2 \cdot q \cdot I_{\text{B}} \cdot A_{\text{b0}} \cdot A_{\text{binf}}}{A_{\text{v}} \cdot A_{\text{i}}}$$

Correlation Admittance



How do we find Y_{corr} ?

$$E(i_{n1} \cdot V_{\text{neq}}) = 0$$

$$i_{n2} = Y_{\text{corr}} \cdot V_{\text{neq}}$$

$$i_{n2} \cdot \overline{V_{\text{neq}}} = Y_{\text{corr}} \cdot V_{\text{neq}} \cdot \overline{V_{\text{neq}}}$$

$$E[(i_n - i_{n1}) \cdot \overline{V_{\text{neq}}}] = Y_{\text{corr}} \cdot E(V_{\text{neq}} \cdot \overline{V_{\text{neq}}})$$

$$Y_{\text{corr}} = \frac{E(i_n \cdot \overline{V_{\text{neq}}}) - E(i_{n1} \cdot \overline{V_{\text{neq}}})}{E(V_{\text{neq}} \cdot \overline{V_{\text{neq}}})} = \frac{E(V_{\text{neq}} \cdot \overline{I_{\text{neq}}})}{V_{\text{neq}}^2}$$

Substitute the following:

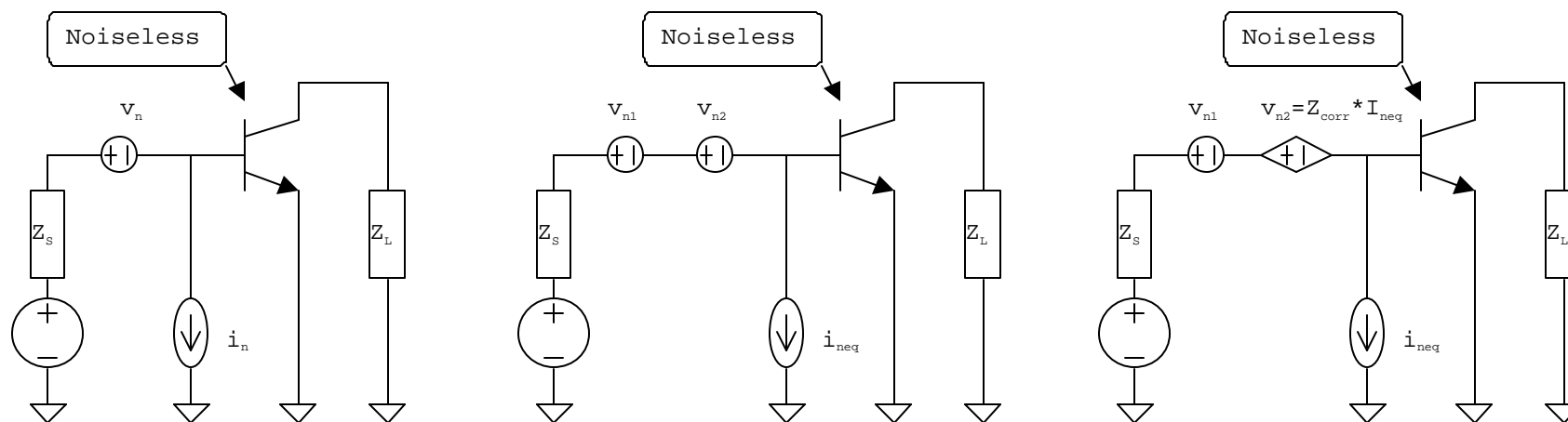
$$E(V_{\text{neq}} \cdot \overline{I_{\text{neq}}}) = g \cdot \sqrt{V_{\text{neq}}^2 \cdot I_{\text{neq}}^2}$$

$$Y_{\text{corr}} = g \cdot \sqrt{\frac{E[(|I_{\text{neq}}|)^2]}{E[(|V_{\text{neq}}|)^2]}}$$

$$G_{\text{corr}} = \text{Re}(Y_{\text{corr}})$$

$$B_{\text{corr}} = \text{Im}(Y_{\text{corr}})$$

Correlation Impedance



How do we find Z_{corr} ?

$$E(V_{n1} \cdot I_{\text{neq}}) = 0$$

$$V_{n2} = Z_{\text{corr}} \cdot I_{\text{neq}}$$

$$V_{n2} \cdot \overline{I_{\text{neq}}} = Z_{\text{corr}} \cdot I_{\text{neq}} \cdot \overline{I_{\text{neq}}}$$

$$E[(V_{\text{neq}} - V_{n1}) \cdot \overline{I_{\text{neq}}}] = Z_{\text{corr}} \cdot E(I_{\text{neq}} \cdot \overline{I_{\text{neq}}})$$

$$Z_{\text{corr}} = \frac{E(V_{\text{neq}} \cdot \overline{i_{\text{neq}}}) - E(i_{n1} \cdot \overline{V_{\text{neq}}})}{E(I_{\text{neq}} \cdot \overline{I_{\text{neq}}})} = \frac{E(V_{\text{neq}} \cdot \overline{I_{\text{neq}}})}{I_{\text{neq}}^2}$$

Substitute the following:

$$E(V_{\text{neq}} \cdot \overline{I_{\text{neq}}}) = g \cdot \sqrt{V_{\text{neq}}^2 \cdot I_{\text{neq}}^2}$$

$$Z_{\text{corr}} = g \cdot \sqrt{\frac{E[(|V_{\text{neq}}|)^2]}{E[(|I_{\text{neq}}|)^2]}}$$

$$R_{\text{corr}} = \text{Re}(Z_{\text{corr}})$$

$$X_{\text{corr}} = \text{Im}(Z_{\text{corr}})$$

Uncorrelated Noise Voltage

r_n is the noise resistance representing the portion of V_{neq}^2 which is uncorrelated with I_{neq}^2 . It is always less than R_n , and in the case where the noise is 100% correlated, it is equal to 0.

$$V_{neq} \cdot \overline{V_{neq}} = (V_{n1} + V_{n2}) \cdot (\overline{V_{n1}} + \overline{V_{n2}})$$

$$E\left[(|V_{neq}|)^2 \right] = E(V_{n1} \cdot \overline{V_{n1}}) + E(V_{n1} \cdot \overline{V_{n2}}) + E(V_{n2} \cdot \overline{V_{n1}}) + E(V_{n2} \cdot \overline{V_{n2}})$$

$$E\left[(|v_{n1}|)^2 \right] = E\left(|V_{neq}^2| \right) - (|Z_{corr}|)^2 \cdot E\left(|I_{neq}^2| \right)$$

$$r_n = \frac{E\left[(|v_{n1}|)^2 \right]}{4 \cdot k \cdot T} = R_n - g_n \cdot (|Z_{corr}|)^2 \quad r_n = R_n \cdot [1 - (|g|)^2]$$

Note: $R_n \ll r_n$!

Uncorrelated Noise Current

G_n is the noise conductance representing the portion of I_{neq}^2 which is uncorrelated with V_{neq}^2 . It is always less than g_n , and in the case where the noise is 100% correlated, it is equal to 0.

$$I_{neq} \cdot \overline{I_{neq}} = (i_{n1} + i_{n2}) \cdot (\overline{i_{n1}} + \overline{i_{n2}})$$

$$E[(|I_{neq}|)^2] = E(i_{n1} \cdot \overline{i_{n1}}) + E(i_{n1} \cdot \overline{i_{n2}}) + E(i_{n2} \cdot \overline{i_{n1}}) + E(i_{n2} \cdot \overline{i_{n2}})$$

$$E[(|i_{n1}|)^2] = E[(|I_{neq}|)^2] - (|Y_{corr}|)^2 \cdot E[(|V_{neq}|)^2]$$

$$G_n = \frac{E[(|i_{n1}|)^2]}{4 \cdot k \cdot T} = g_n - R_n \cdot (|Z_{corr}|)^2$$

$$G_n = g_n \cdot [1 - (|g|)^2]$$

Note: $G_n < g_n$!

Noise Figure

Output-referred noise voltage with arbitrary source impedance Z_S

$$V_o = (V_{n1} + V_{n2} + V_{sn}) \cdot \frac{Z_{in}}{Z_{in} + Z_S} \cdot A + i_n \cdot \frac{Z_{in} \cdot Z_S}{Z_{in} + Z_S} \cdot A$$

Input-referred noise voltage with arbitrary source impedance Z_S .

$$V_{seq} = V_{n1} + V_{n2} + V_{sn} + I_{neq} \cdot Z_S = V_{n1} + I_{neq} \cdot Z_{corr} + V_{sn} + I_{neq} \cdot Z_S$$

$$V_{seq} \cdot \overline{V_{seq}} = [V_{n1} + V_{sn} + I_{neq} \cdot (Z_S + Z_{corr})] \cdot [\overline{V_{n1}} + \overline{V_{sn}} + \overline{I_{neq}} \cdot (\overline{Z_S} + \overline{Z_{corr}})]$$

Input-referred noise power (normalized to 1 ohm)

$$E[(|V_{seq}|)^2] = E[(|V_{n1}|)^2] + E[(|V_{sn}|)^2] + E[(|I_{neq}|)^2] \cdot (|Z_S + Z_{corr}|)^2$$

Noise figure = (Input-referred noise power)/(Source noise power)

$$F = \frac{E[(|V_{seq}|)^2]}{E[(|V_{sn}|)^2]} = 1 + \frac{V_{n1}^2}{V_{sn}^2} + \frac{I_{neq}^2}{V_{sn}^2} \cdot (|Z_S + Z_{corr}|)^2$$

Substitute the following: $V_{sn}^2 = 4 \cdot k \cdot T \cdot R_S$ $I_{neq}^2 = 4 \cdot k \cdot T \cdot g_n$ $V_{n1}^2 = 4 \cdot k \cdot T \cdot r_n$

$$F = 1 + \frac{r_n}{R_S} + \frac{g_n}{R_S} \cdot (|Z_S + Z_{corr}|)^2$$

Optimal Noise Source Impedance

Set Derivatives of F with Respect to X_S and R_S equal to zero and solve for X_S and R_S .

$$F = 1 + \frac{r_n}{R_S} + \frac{g_n}{R_S} \left[(R_S + R_{\text{corr}})^2 + (X_S + X_{\text{corr}})^2 \right]$$

$$\frac{d}{dX_S} F = 0 = X_{\text{Sopt}} + X_{\text{corr}} \quad X_{\text{Sopt}} = -X_{\text{corr}}$$

$$\frac{d}{dR_S} F = \frac{-r_n}{R_{\text{Sopt}}^2} + g_n \left(1 - \frac{R_{\text{corr}}^2}{R_{\text{Sopt}}^2} \right) = 0 \quad R_{\text{Sopt}} = \sqrt{\frac{r_n}{g_n} + R_{\text{corr}}^2}$$

$$Z_{\text{nopt}} = \sqrt{\frac{r_n}{g_n} + R_{\text{corr}}^2} - j \cdot X_{\text{corr}}$$

Optimal Noise Figure

Plug Optimal Noise Source Impedance, Z_{nopt} , into Noise Figure Equation and Solve for F_{min} .

$$F_{\text{min}} = 1 + \frac{r_n}{R_S} + \frac{g_n}{R_S} \cdot \left[\left(\sqrt{\frac{r_n}{g_n} + R_{\text{corr}}^2} + R_{\text{corr}} \right)^2 + (-X_{\text{corr}} + X_{\text{corr}})^2 \right]$$

$$F_{\text{min}} = 1 + 2 \cdot \left[g_n \cdot R_{\text{corr}} + \sqrt{g_n \cdot r_n + (g_n \cdot R_{\text{corr}})^2} \right]$$

Example (Simplified BJT)

$$r_b := 10\Omega$$

Base Resistance

$$b := 100$$

DC Beta

$$I_C := 1\text{mA}$$

Collector Current

$$R_L := 1\text{k}\Omega$$

Load Resistance

$$T := 300\text{K}$$

Temperature

Constants

$$k := 1.3806 \cdot 10^{-23} \cdot \frac{\text{J}}{\text{K}}$$

Boltzman's Constant

$$q := 1.602 \cdot 10^{-19}\text{C}$$

Charge on an Electron

Small Signal Parameters

$$V_T := \frac{k \cdot T}{q}$$

$$V_T = 25.854\text{mV}$$

Thermal Voltage

$$g_m := \frac{I_C}{V_T}$$

$$g_m = 38.679 \frac{\text{mA}}{\text{V}}$$

Device Transconductance

$$I_B := \frac{I_C}{|b|}$$

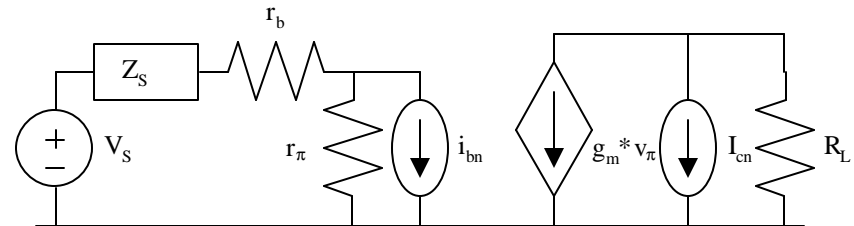
$$I_B = 10\text{mA}$$

Base Current

$$r_p := \frac{b}{g_m}$$

$$r_p = 2.585\text{k}\Omega$$

Base Emitter Resistance



Gain Calculations for Example

$$A_{r0} := \frac{r_p}{r_p + r_b} \cdot g_m \cdot R_L$$

$$A_{r0} = 38.53$$

Base Resistance Gain with $Z_S=0$

$$A_{b0} := \frac{r_b \cdot r_p}{r_p + r_b} \cdot g_m \cdot R_L$$

$$A_{b0} = 385.298 \text{ ohm}$$

Base Current Gain with $Z_S=0$

$$A_{c0} := R_L$$

$$A_{c0} = 1 \text{ kW}$$

Collector Current Gain with $Z_S=0$

$$A_v := \frac{r_p}{r_p + r_b} \cdot g_m \cdot R_L$$

$$A_v = 38.53$$

Input Voltage Gain with $Z_S=0$

$$A_{rinf} := 0$$

$$A_{rinf} = 0$$

Base Resistance Gain with $Z_S=inf.$

$$A_{binf} := r_p \cdot g_m \cdot R_L$$

$$A_{binf} = 100 \text{ kW}$$

Base Current Gain with $Z_S=inf.$

$$A_{cinf} := R_L$$

$$A_{cinf} = 1 \text{ kW}$$

Collector Current Gain with $Z_S=inf.$

$$A_i := r_p \cdot g_m \cdot R_L$$

$$A_i = 100 \text{ kW}$$

Input Current Gain with $Z_S=inf.$

Input Noise Calculations

Equivalent Input Noise Voltage

$$V_{\text{neq}} := \sqrt{4 \cdot k \cdot T \cdot r_b + 2 \cdot q \cdot I_B \cdot r_b^2 + 2 \cdot q \cdot I_C \cdot \left(\frac{1}{g_m} \right)^2 \cdot \left(\left| \frac{r_p + r_b}{r_p} \right| \right)^2}$$
$$V_{\text{neq}} = 0.618 \frac{\text{nV}}{\sqrt{\text{Hz}}}$$

Equivalent Input Noise Resistance

$$R_n := \frac{V_{\text{neq}}^2}{4 \cdot k \cdot T}$$
$$R_n = 23.046 \text{ W}$$

Equivalent Input Noise Current

$$I_{\text{neq}} := \sqrt{2 \cdot q \cdot I_B + \frac{2 \cdot q \cdot I_C}{(|g_m \cdot r_p|)^2}}$$
$$I_{\text{neq}} = 1.799 \frac{\text{pA}}{\sqrt{\text{Hz}}}$$

Equivalent Input Noise Conductance

$$g_n := \frac{I_{\text{neq}}^2}{4 \cdot k \cdot T}$$
$$\frac{1}{g_n} = 5.12 \text{ kW}$$

Noise Correlation

$$V_{I_{\text{conj}}} := \frac{2 \cdot q \cdot I_C \cdot (r_p + r_b)}{(|g_m \cdot r_p|)^2} + 2 \cdot q \cdot I_B \cdot r_b$$

$$V_{I_{\text{conj}}} = 1.152 \times 10^{-22} \frac{\text{W}}{\text{Hz}}$$

$$g := \frac{V_{I_{\text{conj}}}}{\sqrt{V_{\text{neq}}^2 \cdot I_{\text{neq}}^2}}$$

$$g = 0.104 \quad \text{Correlation Coefficient}$$

$$g_{\text{guess}} := \frac{1}{\sqrt{b}}$$

$$g_{\text{guess}} = 0.1 \quad \text{Correlation Guess}$$

($g_m \cdot r_b \ll 1, \beta \gg 1$)

$$Y_{\text{corr}} := g \cdot \frac{I_{\text{neq}}^2}{V_{\text{neq}}^2}$$

$$\frac{1}{Y_{\text{corr}}} = 3.314 \text{ kW} \quad \text{Correlation Conductance}$$

$$Z_{\text{corr}} := g \cdot \frac{V_{\text{neq}}^2}{I_{\text{neq}}^2}$$

$$Z_{\text{corr}} = 35.598 \text{ W} \quad \text{Correlation Impedance}$$

$$r_n := R_n \cdot [1 - (|g|)^2]$$

$$r_n = 22.799 \text{ W} \quad \text{Correlation Resistance}$$

$$G_n := g_n \cdot [1 - (|g|)^2]$$

$$\frac{1}{G_n} = 5.175 \text{ kW} \quad \text{Correlation Conductance}$$

Optimal Noise Figure and Source Impedance

Optimal Source Impedance for Minimal Noise Figure

$$Z_{\text{nopt}} := \sqrt{\frac{r_n}{g_n} + \text{Re}(Z_{\text{corr}})^2} - j \cdot \text{Im}(Z_{\text{corr}})$$

$$Z_{\text{nopt}} = 343.495 \text{ ohm}$$

Optimal Noise Figure

$$\text{NF}_{\text{min}} := 10 \cdot \log \left[1 + 2 \cdot \left[g_n \cdot \text{Re}(Z_{\text{corr}}) + \sqrt{g_n \cdot r_n + (g_n \cdot \text{Re}(Z_{\text{corr}}))^2} \right] \right]$$

$$\text{NF}_{\text{min}} = 0.6 \text{ dB}$$

Noise Figure Plots

$$\text{NF}(Z_S) := 10 \cdot \log \left[1 + \frac{r_n}{\text{Re}(Z_S)} + \frac{g_n}{\text{Re}(Z_S)} \cdot (|Z_S + Z_{\text{corr}}|)^2 \right]$$

$i := 1..20$

$$R_{S_i} := \frac{i}{20} \cdot 600\text{ohm}$$

$$Z_{S_i} := R_{S_i} + j \cdot \text{Im}(Z_{\text{nopt}})$$

Sweeping R_S

$$X_{S_i} := \frac{i}{20} \cdot 400\text{ohm} - 200\text{ohm}$$

$$Z_{S2_i} := \text{Re}(Z_{\text{nopt}}) + j \cdot X_{S_i}$$

Sweeping X_S

