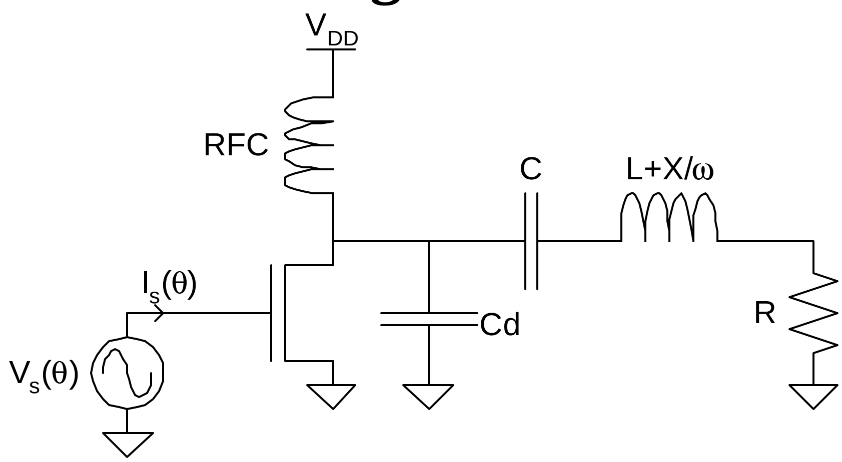
Class E Design Formulas



▶ useful functions and identities
▶ Units
▶ Constants

Table of Contents

Introduction

This file is a modification of the routine class-e1.mcd. The modifications include:
1. The addition of finite Q component losses in the overall efficiency of the amplifier.
2. The effects of a finite device output capacitance. Later the effects of the nonlinearity of this capacitance is also added. This derivation is from reference [1].
3. Units are added.
4. The overall format of the routine is modified for easier use.

A problem with this analysis is that it doesn't take into account overall transmitter chain efficiency due to finite gain loss of the power amplifier. This may set an upper limit on the device size, which will in turn make the drain capacitance part linear capacitance and part nonlinear capacitance. The design also does not take into account the input power back-off required to meet ACPR (adjacent channel power ratio) requirements. This will also degrade efficiency.

Process Parameters

$C_{j0} := 0.4 \frac{fF}{\mu m^2}$	Bottom Plate Capacitance for Zero-Biased Reverse Biased
$C_{jsw} := 0.4 \frac{fF}{\mu m}$	Sidewall Capacitance for Zero-Biased Reverse Biased Dioc
$m_j := 0.77$	Exponent for Reverse Biased Diode Capacitance
$V_{bi} := 1V$	Fermi Potential for Reverse Biased Diode
$\mu_N := 300 \frac{cm^2}{V \cdot sec}$	Device Effective Mobility
$C_{OX} := 2 \frac{fF}{\mu m^2}$	Device Gate Capacitance per Unit Area
$mA\_\mu m := 0.5 \frac{mA}{\mu m}$	Electromigration Limition on Line Width
$L_{min} := 0.5 \mu m$	Minimum Line Width
$V_{TN} := 0.7V$	Threshold Voltage

Inputs

$V_{dd} := 2V$	Supply Voltage
$V_{sat} := 0.2V$	Minimum Device Drain to Source Voltage
$P_a := 1.25 watt$	Power Available at Drain
$Q := 7$	Q of Matching Network (Optimum Value can be found: Higher value leads to narrower band and more dependence on process variations and finite Q elements. Lower value degrades efficiency directly)
$f := 0.9 \cdot GHz$	Center Frequency
$t_r := 0.1 \cdot nS$	Rise Time

Standard Class-E Synthesis Procedure

$\omega := 2 \cdot \pi \cdot f$	$\omega = 5.655 \frac{Grad}{sec}$	Center Frequency
$R_s := \frac{0.577 \cdot (V_{dd} - V_{sat})^2}{P_a}$	$R_s = 1.496\,ohm$	Device Load Resistance
$L_s := \frac{Q \cdot R_s}{\omega}$	$L_s = 1.851\,nH$	Matching Inductance
$C_p := \frac{1}{\omega \cdot R_s \cdot 5.447}$	$C_p = 21.707\,pF$	Matching Capacitances
$C_s := C_p \cdot \left(\frac{5.447}{Q}\right) \cdot \left(1 + \frac{1.42}{Q - 2.08}\right)$	$C_s = 21.767\,pF$	
$V_p := 3.562 \cdot V_{dd} - 2.562 \cdot V_{sat}$	$V_p = 6.612\,V$	Peak Output Voltage

$$A := \left(1 + \frac{\quad}{Q}\right)\cdot f\cdot t_r \qquad A = 0.101$$

$$I_{dc} := \frac{P_a}{V_{dd}}\cdot\frac{1 - \frac{(2\cdot\pi\cdot A)^2}{12}}{1 - \frac{(2\cdot\pi\cdot A)^2}{6} - \left(\frac{V_{sat}}{V_{dd}}\right)\left[1 + A - \frac{(2\cdot\pi\cdot A)^2}{6}\right]} \qquad I_{dc} = 727.896\,\text{mA}$$ DC Current

$$I_p := I_{dc}\cdot\left[1 + 1.862\cdot\left(1 - \frac{0.5}{Q}\right)\right] \qquad I_p = 1.986\,\text{amp}$$ Peak Current

$$P_{dc} := V_{dd}\cdot I_{dc} \qquad P_{dc} = 1.456\,\text{watt}$$ DC Power

$$\eta := \frac{P_a}{P_{dc}}\cdot 100 \qquad \eta = 85.864$$ Efficiency

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Alinikula's Class E Design Analyis and Synthesis

Solution Assuming a Linear Capacitor

$$n := \frac{m_j}{1 - m_j} \qquad n = 3.348$$ Substitute parameter for mj

$$C_D := \frac{P_a}{\pi\cdot\omega\cdot V_{dd}^{\,2}} \qquad C_D = 17.59\,\text{pF}$$ Device Output Capacitance

$$R := \frac{8}{\pi^2 + 4}\cdot\frac{V_{dd}^{\,2}}{P_a} \qquad R = 1.846\,\Omega$$ Load Resistance

$$X := \frac{\pi\cdot\left(\pi^2 - 4\right)}{2\cdot\left(\pi^2 + 4\right)}\cdot\frac{V_{dd}^{\,2}}{P_a} \qquad X = 2.127\,\Omega$$ Load Reactance

$$v_{Dmax} := 2\cdot\pi\cdot\text{atan}\left(\frac{2}{\pi}\right)\cdot V_{dd} \qquad v_{Dmax} = 7.124\,\text{V}$$ Maximum Drain Voltage

$$\eta := \frac{R}{R + 1.365\frac{V_{sat}}{I_{dc}}} \qquad \eta = 0.831$$ Drain Efficiency

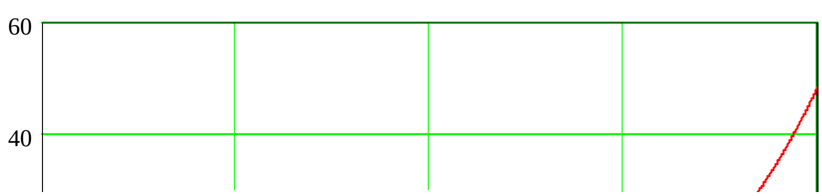
Solution Assuming a Nonlinear Capacitor

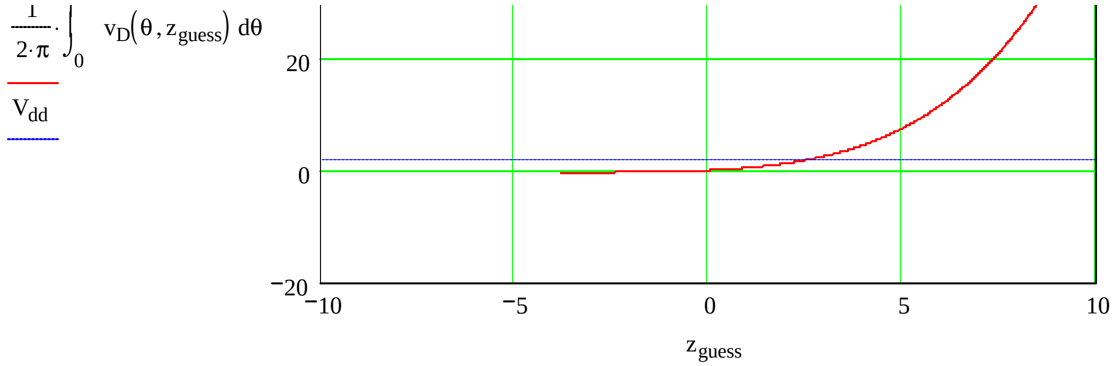
$$v_D(\theta) = V_{bi}\cdot\left[\left[\frac{1}{\omega\cdot C_{j0}\cdot V_{bi}\cdot(n+1)}\cdot\left(I_{DC}\cdot\theta + \frac{v_{out}}{R}\cdot\cos(\theta+\phi) - \cos(\phi)\right) + 1\right]^{n+1} - 1\right]$$

$$v_D(\theta) = V_{bi}\cdot\left[\left[\frac{z}{(n+1)}\cdot\left[\theta + \frac{v_{out}}{R\cdot I_{DC}}\cdot\left(\cos(\theta+\phi) - \cos(\phi)\right)\right] + 1\right]^{n+1} - 1\right]$$

$$v_D(\theta, z) := V_{bi}\cdot\left[\left[\frac{z}{n+1}\cdot\left(\theta + \frac{\pi}{2}\cdot\cos(\theta) + \sin(\theta) - \frac{\pi}{2}\right) + 1\right]^{n+1} - 1\right]$$ Drain Voltage

Using Root Finder to solve for z





$$z := \begin{array}{|l} I_{DCguess} \leftarrow \dfrac{V_{dd}}{R} \\ z_{guess} \leftarrow \dfrac{I_{DCguess}}{\omega \cdot C_D \cdot V_{bi}} \\ \text{root}\left(V_{dd} - \dfrac{1}{2\cdot\pi}\cdot\int_0^{\pi} v_D(\theta, z_{guess})\, d\theta, z_{guess}\right) \end{array}$$

$z = 2.642$ Variable used for more efficient calculation.

$$\phi_1 := \text{atan}\left(\frac{\int_0^{\pi} v_D(\theta, z)\cdot\cos(\theta)\, d\theta}{\int_0^{\pi} v_D(\theta, z)\cdot\sin(\theta)\, d\theta} \right)$$

$\phi_1 = 18.922\,\text{deg}$ Fundamental Frequency Phase Angle at node A

$$a_1 := \frac{1}{\pi}\cdot\sqrt{\left(\int_0^{\pi} v_D(\theta, z)\cdot\cos(\theta)\, d\theta\right)^2 + \left(\int_0^{\pi} v_D(\theta, z)\cdot\sin(\theta)\, d\theta\right)^2}$$

$a_1 = 3.443\,\text{V}$ Fundamental Frequency Amplitude at node A

$$R := \frac{8}{\pi^2 + 4}\cdot\frac{V_{dd}^{\ 2}}{P_a}$$

$R = 1.846\,\text{ohm}$ Load Resistance

$$X := R\cdot\tan\left(\phi_1 - \text{atan}\left(\frac{-2}{\pi}\right)\right)$$

$X = 2.312\,\Omega$ Load Reactance

$$C_j := \frac{3.562\cdot V_{dd}}{z\cdot 2\cdot\text{atan}\left(\frac{2}{\pi}\right)\cdot V_{bi}}\cdot C_D$$

$C_j = 41.837\,\text{pF}$ Drain Capacitance

$$I_{DC} := z\cdot\omega\cdot C_j\cdot V_{bi}$$

$I_{DC} = 624.998\,\text{mA}$ DC Current

$$L := \frac{X}{\omega}$$

$L = 0.409\,\text{nH}$ Load Inductance (if X is positive)

$$C := \frac{-1}{\omega\cdot X}$$

$C = -76.473\,\text{pF}$ Load Capacitance (if X is negative)

$$C_{res} := \frac{1}{Q\cdot\omega\cdot R}$$

$C_{res} = 13.687\,\text{pF}$ Resonant Capacitance

$$L_{res} := \frac{Q\cdot R}{\omega}$$

$L_{res} = 2.285\,\text{nH}$ Resonant Inductance

$$v_{Dmax} := V_{bi}\cdot\left[\left[\frac{z}{n+1}\cdot\left(-2\cdot\text{atan}\left(\frac{-2}{\pi}\right)\right) + 1\right]^{n+1} - 1\right]$$

$v_{Dmax} = 8.764\,\text{V}$ Maximum Drain Voltage

I_{DC} 1

$N := \frac{I_{DC}}{mA\_\mu m} \cdot \frac{1}{3 \cdot L_{min}}$ $N = 833.331$ Number of Devices in Parallel

The following equation for the drain capacitance is based on a layout, where the drain is layed out

$$C_j = N \cdot \frac{C_{j0} \cdot 3 \cdot L_{min} \cdot W_0 + C_{jsw} \cdot 2 \cdot (W_0 + 3 \cdot L_{min})}{2}$$

$W_0 := \frac{\frac{C_j \cdot 2}{N} - 3 \cdot L_{min} \cdot C_{jsw} \cdot 2}{C_{j0} \cdot 3 \cdot L_{min} + C_{jsw} \cdot 2}$ $W_0 = 70.863\,\mu m$ Width of Each Segment

$W := W_0 \cdot N$ $W = 59.053\,mm$ Width of Overall Device

$Length := L_{min} + 3 \cdot L_{min} \cdot N$ $Length = 1.25\,mm$ Length of N Devices

$Area := W_0 \cdot 4 \cdot L_{min} \cdot N$ $Area = 0.118\,mm^2$ Overall Area of Device

$R_{on} := \frac{1}{\mu_N \cdot C_{OX} \cdot (V_{dd} - V_{TN}) \cdot \frac{W}{L_{min}}}$ $R_{on} = 0.109\,\Omega$ On Resistance of Device

$\eta := \frac{R}{R + 1.365 R_{on}}$ $\eta = 92.569\,\%$ Drain Efficiency

Choi's Class E Design Procedure

Notes about this design procedure:

1. It does not include the effects of inductive source degeneration, but instead models it as a resistive source degeneration. The paper does not say what the degeneration is, but it implies it is the same as the gate impedance
2. It does not calculate the input impedance, which is necessary for matching to the driver amplifier. This impedance changes during the cycle, so what value do you use for the input matching network? The average value?
3. It does not calculate the required input signal swing, Vs, to generate the required output power. It is possible to use the equations in reverse to find the required Vs, but will surely be non-linear, so iterative technques must be used. Since iterative techniques must be used it might be more appropriate not to solve the equations in reverse, but instead use constraint-based optimization to meet the two conditions of output power and maximize PAE.
4. It does not include parasitic losses of the output matching network. This should be a simple addition.

$\frac{\omega}{2 \cdot \pi} = 900\,MHz$ Center frequency of oscillator

$R := 50\,ohm$

$P_{out} := P_a$ $P_a = 1.25\,watt$ Designed output power at the drain.

$P_{out} := 1\,watt$

$C_d := \frac{P_{out}}{\pi \cdot \omega \cdot V_{dd}^2}$ $C_d = 14.072\,pF$ Drain capacitance

$\mu_N := 546 \frac{cm^2}{V \cdot sec}$ Device mobility

$K_n := 200 \frac{\mu A}{V^2}$ Instrinsic transconductance

$sq := 1$

$gatefinger := 1$

$r_m := 0.07 \frac{\Omega}{}$ Metal resistance sq

$r_c := 1.8 \frac{\Omega}{gatefinger}$ Metal-poly contact resistance

$R_s(n) := r_m + \frac{r_c}{n}$ Resistance in the source $R_s(n) = 0.608\,\Omega$

$\rho_{gate} := 1.9 \frac{\Omega}{sq}$ Polysilicon sheet resistance

$C_{GD0} := 3 \cdot 10^{-10} \frac{farad}{m}$ Gate to drain overlapp capacitance

$W := 125\mu m$ Width of one finger

$L := 0.5\mu m$ Length of one finger

$r_g := \left(\frac{\rho_{gate}}{3} \cdot \frac{W}{L} + r_c \right)$ $r_g = 160.133\,\Omega$ $R_g(n) := r_m + \frac{r_g}{n}$ Gate resistance

$c_{gs} := \frac{K_n \cdot W \cdot L}{\mu_N}$ $c_{gs} = 0.229\,pF$ $C_{gs}(n) := n \cdot c_{gs}$ Gate to source capacitance

$c_{gd0} := C_{GD0} \cdot W$ $c_{gd0} = 0.038\,pF$ $C_{gdo}(n) := n \cdot c_{gd0}$ Gate to drain capacitance

$$C_{gd}(n) := C_{gdo}(n) + \frac{C_{gs}(n)}{2}$$

$$I_d(\pi) = \frac{K_n \cdot W}{2 \cdot L} \cdot \left(2 \cdot V_{gs}(\pi) \cdot V_{ds}(\pi) - V_{ds}(\pi)^2 \right) = 2 \cdot I_{DC}$$

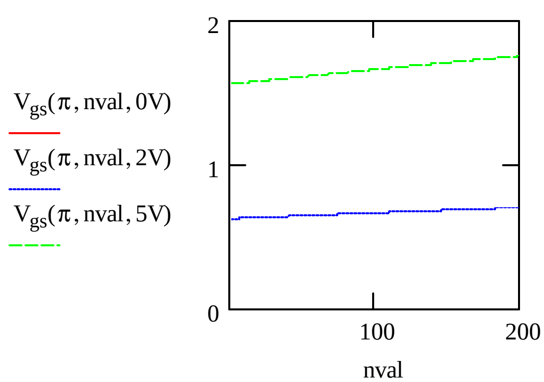
$V_{sm} := 4.3V$

$$V_{gsmag}(n, V_{sm}) := \frac{V_{sm}}{\sqrt{1 + \left[\omega \cdot \left(R_g(n) + R_s(n)\right) \cdot \left(C_{gs}(n) + C_{gd}(n)\right) \right]^2}}$$ $V_{gsmag}(n, V_{sm}) = 4.059\,V$

$\chi(n) := atan\left[-\omega \cdot \left(R_g(n) + R_s(n)\right) \cdot \left(C_{gs}(n) + C_{gd}(n)\right) \right]$ $\chi(n) = -0.337$

$V_{gs}(\theta) := V_{gsmag}(n) \cdot \sin(\theta + \chi(n)) - I_d(\theta) \cdot R_s(n)$

$V_{gs}(\theta, n, V_{sm}) := V_{gsmag}(n, V_{sm}) \cdot \sin(\theta + \chi(n))$ $V_{gs}(\pi, n, V_{sm}) = 1.34\,V$



$$2 \cdot \pi \cdot \omega \cdot R_s(n) \cdot C_d \cdot \left(V_{dd} - V_{gs}(\pi, n, V_{sm})\right) - 2 \cdot \pi \cdot \omega \cdot C_d \cdot \frac{L}{K_n \cdot W} - V_{gs}(\pi, n, V_{sm}) = -11.14\,kg\,m^2\,s^{-3}\,A^{-1}$$

$a(n) := 1 - 2 \cdot \pi \cdot \omega \cdot C_d \cdot R_s(n)$ $a(n) = 0.696$

$1 + 2\cdot\pi\cdot\omega\cdot R_s(n)\cdot C_d = 1.304$

$a(n) := 1 - 2\cdot\pi\cdot\omega\cdot R_s(n)\cdot C_d$ $\qquad a(n) = 0.696$

$$b(n, V_{sm}) := 2\cdot\left[2\cdot\pi\cdot\omega\cdot R_s(n)\cdot C_d\cdot\left(V_{dd} - V_{gs}(\pi, n, V_{sm})\right) - 2\cdot\pi\cdot\omega\cdot C_d\cdot\frac{L}{K_n\cdot W} - V_{gs}(\pi, n, V_{sm})\right]$$

$b(n, V_{sm}) = -22.279\,V$

$$b(n, V_{sm}) := -2\cdot V_{gsmag}(n, V_{sm})\cdot\sin(\pi + \chi(n))\cdot\left(1 + 2\cdot\pi\cdot\omega\cdot R_s(n)\cdot C_d\right) + 4\cdot\pi\cdot\omega\cdot C_d\cdot V_{dd}\cdot R_s(n) - \frac{4\cdot\pi\cdot\omega}{K_n\cdot\frac{W}{L}}\cdot C_d$$

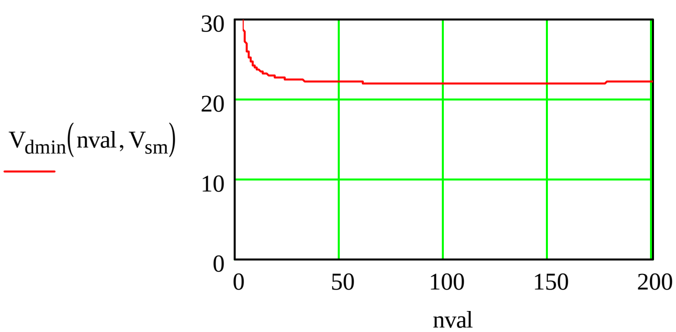
$b(n, V_{sm}) = -22.279\,kg\,m^2\,s^{-3}\,A^{-1}$

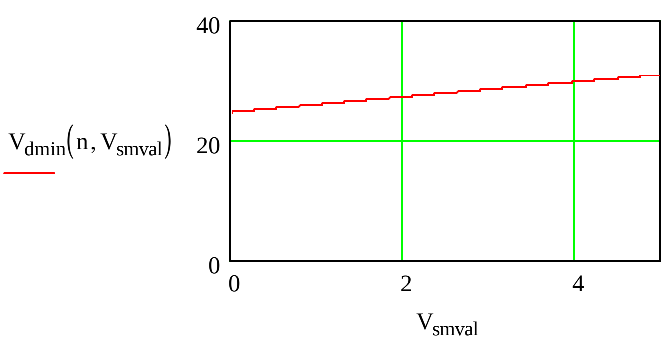
$c(n) := 4\cdot\pi\cdot\omega\cdot C_d\cdot\frac{L}{K_n\cdot W}\cdot V_{dd}$ $\qquad c(n) = 40\,V^2$

$c(n) := \frac{4\cdot\pi\cdot\omega\cdot C_d}{K_n\cdot\frac{W}{L}}\cdot V_{dd}$ $\qquad c(n) = 40\,V^2$

$$V_{dmin}(n, V_{sm}) := \frac{-b(n, V_{sm}) + \sqrt{b(n, V_{sm})^2 - 4\cdot a(n)\cdot c(n)}}{2\cdot a(n)}$$

$V_{dmin}(n, V_{sm}) = 30.094\,V$





$V_{dmin}(n, V_{sm}) := 0.05V$

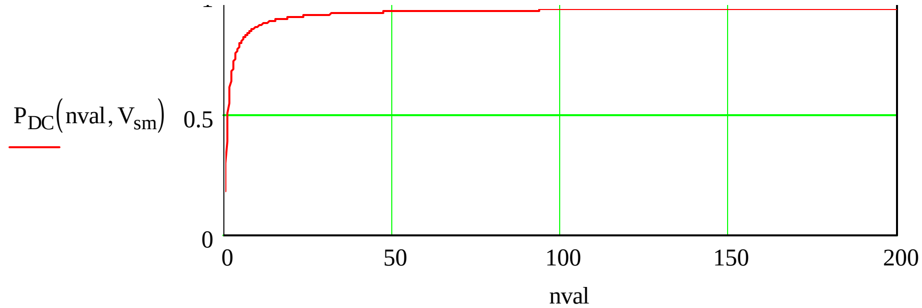
$$I_{DC}(n, V_{sm}) := \frac{\pi\cdot\omega\cdot C_d\cdot\left(V_{dd} - V_{dmin}(n, V_{sm})\right)}{1 + 2\cdot\pi\cdot\omega\cdot R_s(n)\cdot C_d}$$

$I_{DC}(n, V_{sm}) = 0.374\,amp$

$P_{DC} = I_{DC}\cdot V_{DD}$

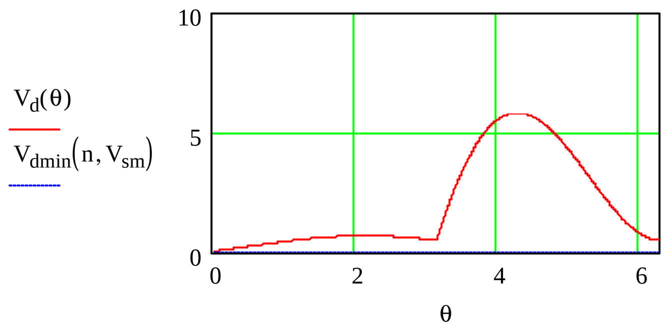
$$P_{DC}(n, V_{sm}) := \frac{\pi\cdot\omega\cdot C_d\cdot\left(V_{dd} - V_{dmin}(n, V_{sm})\right)}{1 + 2\cdot\pi\cdot\omega\cdot R_s(n)\cdot C_d}\cdot V_{dd}$$

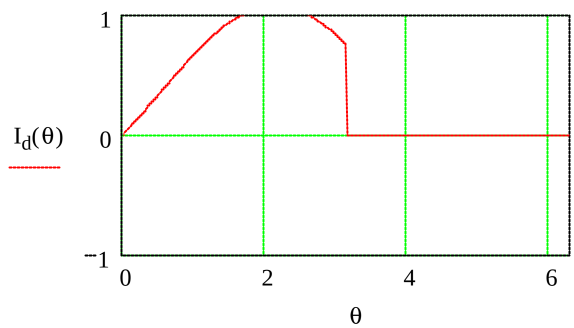
$P_{DC}(n, V_{sm}) = 0.748\,watt$



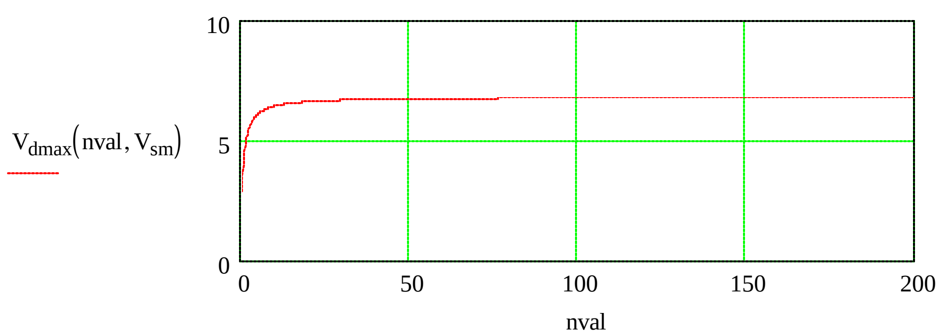
$$V_d(\theta) := \text{if}\left[\theta < \pi, I_{DC}(n, V_{sm}) \cdot R_s(n) \cdot \left(1 + \frac{\pi}{2} \cdot \sin(\theta) - \cos(\theta)\right) \ldots + V_{dmin}(n, V_{sm}), \frac{I_{DC}(n, V_{sm})}{\omega \cdot C_d} \cdot \left(\theta - \frac{3 \cdot \pi}{2} - \frac{\pi}{2} \cdot \cos(\theta) - \sin(\theta)\right) \ldots + V_{dmin}(n, V_{sm}) + 2 \cdot I_{DC}(n, V_{sm}) \cdot R_s(n)\right]$$

$$I_d(\theta) := \text{if}\left[\theta < \pi, I_{DC}(n, V_{sm}) \cdot \left(1 + \frac{\pi}{2} \cdot \sin(\theta) - \cos(\theta)\right), 0V\right]$$





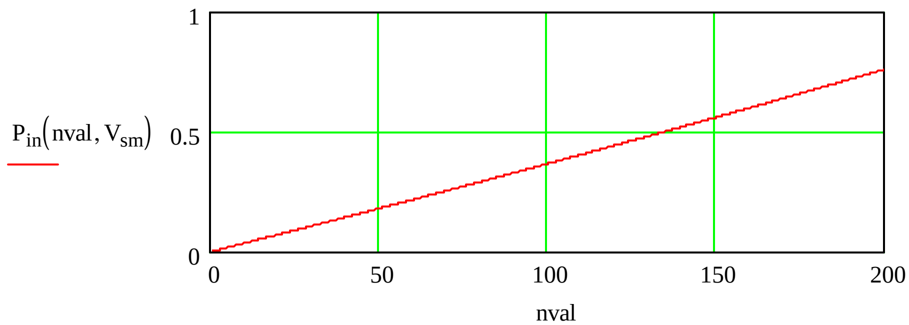
$$V_{dmax}(n, V_{sm}) := 1.13 \cdot \frac{I_{DC}(n, V_{sm})}{\omega \cdot C_d} + V_{dmin}(n, V_{sm}) + 2 \cdot I_{DC}(n, V_{sm}) \cdot R_s(n)$$



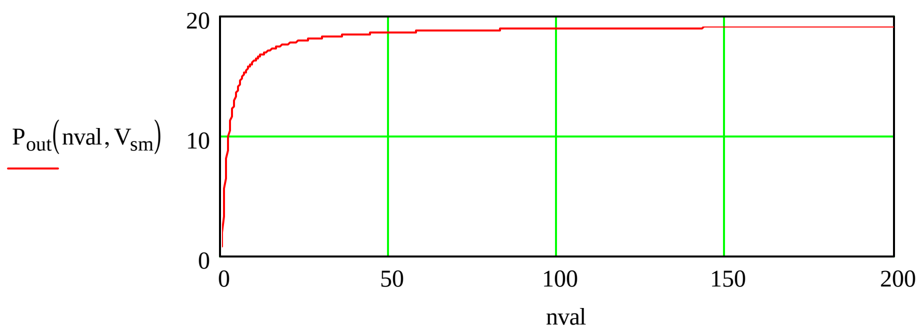
$\omega^2 \cdot V_{sm}$ $r_g \cdot (c_{gs} + c_{gd0})$

$$P_{in}(n, V_{sm}) := \frac{\ldots}{4}\cdot\frac{\ldots}{1 + \left[\omega\cdot r_g\cdot\left(c_{gs} + c_{gd0}\right)\right]^2}\cdot\left[2\cdot V_{sm}\cdot\left(c_{gs} + c_{gd0}\right) + V_{dmax}(n, V_{sm})\cdot c_{gd0}\right]$$

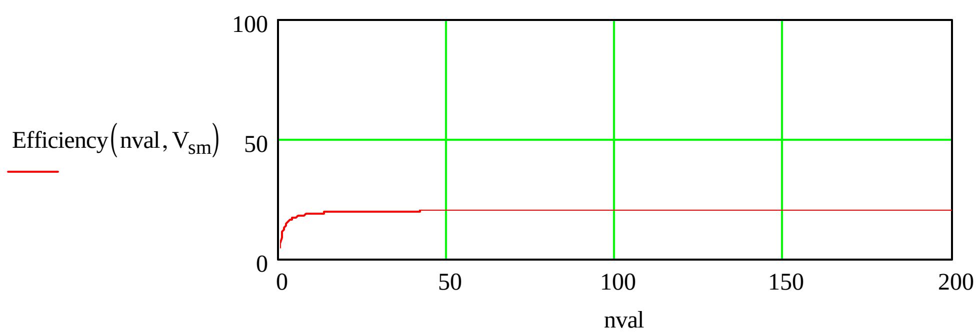
$$P_{in}(n, V_{sm}) := \frac{\omega^2\cdot V_{sm}}{4}\cdot\frac{n\cdot\left(n\cdot r_m + r_g\right)\cdot\left(c_{gs} + c_{gd0}\right)}{1 + \omega^2\cdot\left(n\cdot r_m + r_g\right)^2\cdot\left(c_{gs} + c_{gd0}\right)^2}\cdot\left[2\cdot V_{sm}\cdot\left(c_{gs} + c_{gd0}\right) + V_{dmax}(n, V_{sm})\cdot c_{gd0}\right]$$



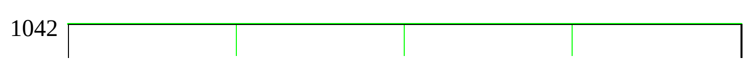
$$P_{out}(n, V_{sm}) := \frac{\pi^2 + 4}{8}\cdot I_{DC}(n, V_{sm})^2\cdot R\cdot\left[1 + \left(\frac{\omega\cdot R_s(n)\cdot C_d}{2}\right)^2\right]$$

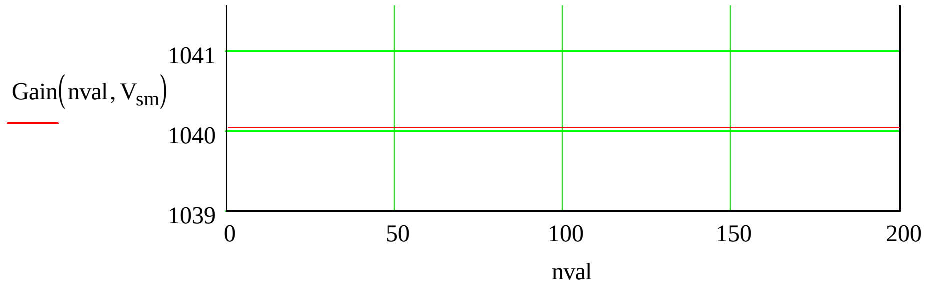


$$\mathrm{Efficiency}(n, V_{sm}) := \frac{I_{DC}(n, V_{sm})\cdot R\cdot\left(\pi^2 + 4\right)\cdot\left[1 + \left(\frac{\omega\cdot R_s(n)\cdot C_d}{2}\right)^2\right]}{8\cdot V_{dd}}$$



$$\mathrm{Gain} = \frac{P_{out}}{P_{in}} \qquad \mathrm{Gain}(nval, V_{sm}) := \frac{I_{DC}(n, V_{sm})^2\cdot R\cdot\left(\pi^2 + 4\right)\cdot\left[1 + \left(\frac{\omega\cdot R_s(n)\cdot C_d}{2}\right)^2\right]\cdot\left[1 + \left[\omega\cdot R_g(n)\cdot\left(C_{gs}(n) + C_{gdo}(n)\right)\right]\right]}{2\cdot V_{sm}\cdot\omega^2\cdot R_g(n)\cdot\left(C_{gs}(n) + C_{gdo}(n)\right)\cdot\left[2\cdot V_{sm}\cdot\left(C_{gs}(n) + C_{gdo}(n)\right) + V_{dmax}(n, V_{sm})\cdot C_{gdo}\right.}$$





$$PAE(n, V_{sm}) := Efficiency - \frac{P_{in}}{P_{DC}}$$

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