Noise Figure Increase due to Mixing of Bias Noise with Jammer

A simple example of the current noise output from a MOSFET driven in saturation with a large sinusoid around a DC bias. The analysis will be extended and used to analyze and design both bipolar and MOS oscillators.

Model Parameters

Device mobility under bias

Gate oxide thickness

Relative permittivity of silicon dioxide

Permittivity of free space

\[ C_{OX} = 3.453 \, \frac{\text{fF}}{\mu \text{m}^2} \]

\[ \mu \cdot C_{OX} = 103.594 \, \frac{\mu \text{A}}{\sqrt{\text{V}}} \]

Gate threshold voltage

Operating Temperature

Boltzmann’s constant
Substitution

Trignometric substitution:

Expand and drop the $v_n(t)^2$ term (noise square is negligible)

Expansion of current noise:

Calculations

Inputs

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{GS}$</td>
<td>1.5V</td>
</tr>
<tr>
<td>$f_0$</td>
<td>1GHz</td>
</tr>
<tr>
<td>$N$</td>
<td>100</td>
</tr>
<tr>
<td>$A$</td>
<td>200 mV</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>DC gate to source bias voltage</td>
<td></td>
</tr>
<tr>
<td>Frequency of input sine wave</td>
<td></td>
</tr>
<tr>
<td>Number of points in time vector</td>
<td></td>
</tr>
<tr>
<td>Amplitude of signal swing at gate</td>
<td></td>
</tr>
</tbody>
</table>

$$t := \frac{2}{f_0 \cdot N \cdot f_0 \cdot N} \cdot \frac{2}{f_0}$$

$$v_g(t) := A \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t)$$

$$I_D(t) := \frac{\mu \cdot C_{OX}}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T + v_g(t))^2$$

$$g_m(t) := \sqrt{2 \cdot \mu \cdot C_{OX} \cdot \frac{W}{L} \cdot I_D(t)}$$

$$g_m(t) := \mu \cdot C_{OX} \cdot \frac{W}{L} \cdot (V_{GS} - V_T - v_g(t))$$

$$v_n(t) := 4 \cdot k \cdot T \cdot \gamma \cdot \frac{1}{g_m(t)}$$

$$I_{spn}(t) := \frac{\mu \cdot C_{OX}}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T + v_g(t) + v_n(t))^2$$

**Expansion of current noise:**

$$I_{spn}(t) := I_0 \cdot \left[ 1 + \frac{A}{V_{Dsat}} \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t) + \frac{4 \cdot k \cdot T \cdot \gamma}{V_{Dsat}} \cdot \frac{1}{\mu \cdot C_{OX} \cdot \frac{W}{L} \cdot 4 \cdot k \cdot T \cdot \gamma} \cdot \left( 1 + \frac{A}{V_{Dsat}} \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t) \right) \right]^2$$

$$I_0 := \frac{\mu \cdot C_{OX}}{2} \cdot \frac{W}{L} \cdot (V_{GS} - V_T)^2$$

$$V_{Dsat} := V_{GS} - V_T$$

$$I_{spn}(t) := I_0 \cdot \left[ 1 + \frac{A}{V_{Dsat}} \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t) + \frac{v_n(t)}{V_{Dsat}} \right]^2$$

Expand and drop the $v_n(t)^2$ term (noise square is negligible)

$$I_{spn}(t) := I_0 \cdot \left[ 1 + 4 \cdot \frac{A}{V_{Dsat}} \cdot \sin(\pi \cdot f_0 \cdot t) \cdot \cos(\pi \cdot f_0 \cdot t) + 2 \cdot \frac{v_n(t)}{V_{Dsat}} \right]$$

And

$$\sin^2(2 \cdot \pi \cdot f_0 \cdot t) = \frac{1}{2} \cdot (1 - \cos(2 \cdot \pi \cdot f_0 \cdot t))$$

Trignometric substitution:

$$\sin(\pi \cdot f_0 \cdot t) \cdot \cos(\pi \cdot f_0 \cdot t) = \frac{1}{2} \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t)$$

$$I_{spn}(t) := I_0 \cdot \left[ 1 + 2 \cdot \frac{A}{V_{Dsat}} \cdot \sin(2 \cdot \pi \cdot f_0 \cdot t) + 2 \cdot \frac{v_n(t)}{V_{Dsat}} + \frac{A^2}{V_{Dsat}^2} \cdot \left( \frac{1}{2} \cdot (1 - \cos(2 \cdot \pi \cdot f_0 \cdot t)) \right) \right]$$
Substitution

\[ g_{m0} \approx \frac{2I_0}{V_{Dsat}} \]

\[ I_{spn}(t) := I_0 \left( 1 + \frac{A^2}{V_{Dsat}} \right) + \left( g_{m0} A \sin(2 \pi f_0 t) \right) - \frac{I_0 A^2}{V_{Dsat}} \frac{1}{2} \cos(2 \pi 2 f_0 t) \ldots \]

\[ + g_{m0} v_n(t) + g_{m0} A \sin(2 \pi f_0 t) \frac{v_n(t)}{V_{Dsat}} \]

This expression represents both the large signal currents and the noise currents. The large signal current by itself is:

\[ I_D(t) := I_0 \left( 1 + \frac{A^2}{V_{Dsat}} \right) + \left( g_{m0} A \sin(2 \pi f_0 t) \right) - \left( \frac{I_0 A^2}{V_{Dsat}} \frac{1}{2} \cos(2 \pi 2 f_0 t) \right) \]

which consists of three components: A DC (or time average) component, the fundamental input component times the small signal transconductance and a second harmonic distortion component. The noise current terms are time varying are represented below

\[ I_n(t) := g_{m0} v_n(t) + g_{m0} A \sin(2 \pi f_0 t) \frac{v_n(t)}{V_{Dsat}} \]

This equation contains two components. The first is a linear cyclostationary white noise component. If viewed under a spectrum analyzer at a rate much less than the input oscillation frequency, the time vary component averages out and the DC component is left

\[ g_{m}(t) := \frac{2 \mu C_{OX} W}{L} I_D(t) \]

Time varying small signal transconductance

\[ g_{m}(t) := \mu C_{OX} \frac{W}{L} \left( V_{GS} - V_T - A \sin(2 \pi f_0 t) \right) \]

Input referred device thermal noise

\[ v_n(t) := 4kT \gamma \frac{1}{g_{m}(t)} \]

\[ v_n(t) := 4kT \gamma \frac{1}{C_{OX} \frac{W}{L}} \left( V_{GS} - V_T - A \sin(2 \pi f_0 t) \right) \]

The time average voltage is

\[ v_{nave} := 4kT \gamma \frac{1}{C_{OX} \frac{W}{L}} \int_0^{2\pi} \frac{1}{(V_{GS} - V_T - A \sin(x))} \, dx \]

\[ v_{nave} := 4kT \gamma \frac{2\pi}{C_{OX} \frac{W}{L}} \left( \frac{\text{csgn} \left( V_{GS} - V_T \right) \sqrt{ \left( V_{GS} - V_T \right)^2 - A^2} }{\sqrt{ \left( V_{GS} - V_T \right)^2 - A^2} } \right) \]

\[ v_{nave} := \frac{4kT \gamma}{C_{OX} \frac{W}{L}} \left( V_{GS} - V_T \right) \]

Or better written as the output referred current noise:

\[ i_{n}(t) := 4kT \gamma \mu C_{OX} \frac{W}{L} \left( V_{GS} - V_T - A \sin(2 \pi f_0 t) \right) \]

The time average of this is

\[ i_{nave} := 4kT \gamma \mu C_{OX} \frac{W}{L} \left( V_{GS} - V_T \right) \]

\[ v_{nave} = 4kT \gamma g_{m0} + \frac{K}{f} \]

which is the same without the large signal input. The last noise term represents a mixing term:

\[ \cos(2 \pi f_1) \cos(2 \pi f_2) = \frac{1}{2} \left( \cos(f_1 + f_2) + \cos(f_1 - f_2) \right) \]
\[
g_{m0} A \cdot \sin(2 \pi f_0 t) \frac{v_n(t)}{V_{Dsat}} \]

If the noise is white than noise from one location replaces noise from another location and the resultant is a white noise floor. Since noise is uncorrelated from one frequency point to the next it will add uncorrelated to the first component of noise

\[
in_{\text{white}}(f) := 4 k T \cdot \gamma \cdot g_{m0} \left( 1 + \frac{A}{V_{Dsat}} \right)
\]

But, any low frequency noise components such as 1/f noise will noise mix around the carrier. 1/2 will go to one side of the carrier and 1/2 will go to the other side of the carrier

\[
in_{1/f}(f) := g_{m0} \frac{A}{V_{Dsat}} \left[ \frac{K}{2(f + f_0)} + \frac{K}{2(f - f_0)} \right]
\]

The net result is:

1. The output current increases by \((1 + A^2/V_{Dsat})^2/2\)
2. The output white current noise increases by \((1 + A/V_{Dsat})\)
3. The output 1/f noise is simply \(g_{m0} A f\)
4. Two new 1/f terms are added: \(g_{m0} A V_{Dsat} K/2 ((1/(f - f_0) + 1/(f - f_0))\)
5. The fundamental exists: \(g_{m0} v_i\)
6. A second harmonic exists: \(g_{m0} A V_{Dsat} / 2 v_i\)

Copyright and Trademark Notice

All software and other materials included in this document are protected by copyright, and are owned or controlled by Circuit Sage.

The routines are protected by copyright as a collective work and/or compilation, pursuant to federal copyright laws, international conventions, and other copyright laws. Any reproduction, modification, publication, transmission, transfer, sale, distribution, performance, display or exploitation of any of the routines, whether in whole or in part, without the express written permission of Circuit Sage is prohibited.