

Sallen-Key Low Pass Filter Design Routine

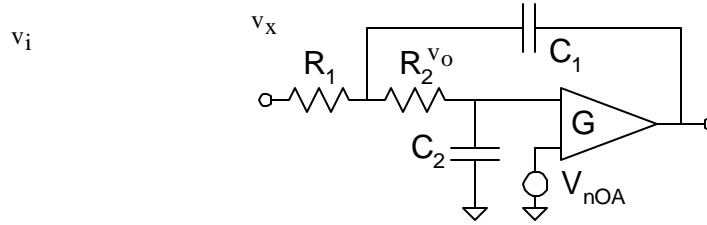


Fig. 1: Single-ended Sallen-Key filter

Note: This routine is a reduction of a more complex version. This reduction is still taking place, so please excuse the current mess.

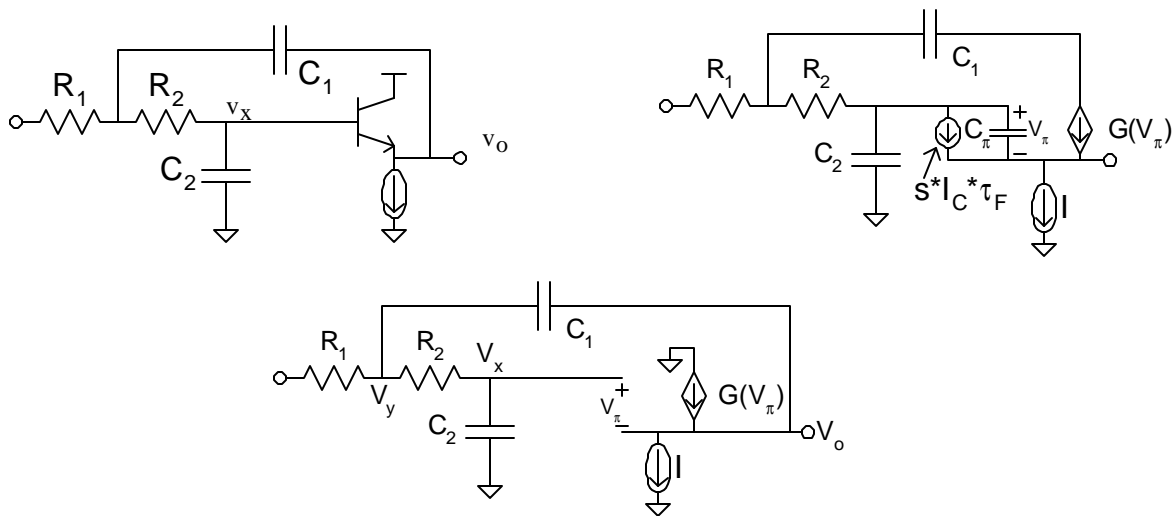


Fig. 2: Single-Ended Sallen-Key Filter w/ Emitter Follower

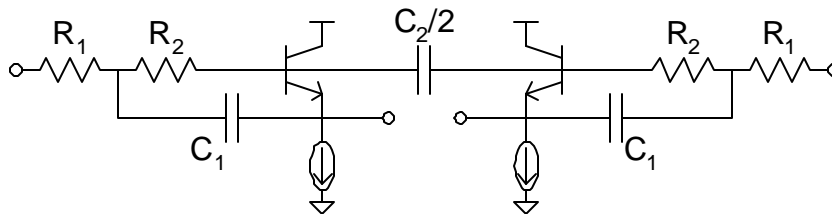


Fig. 3: Differential Voltage-Driven Sallen-Key Filter

useful functions and identities

Units

Constants

Units/Constants/Model File

$$V_o = \frac{V_i}{\left[1 + (R_2 + R_1) \cdot C_2 \cdot s + R_1 \cdot R_2 \cdot C_1 \cdot C_2 \cdot s^2 \right]} = \frac{1}{\left(\frac{s}{\omega_0} \right)^2 + \frac{s}{\omega_0} \cdot \frac{1}{Q} + 1}$$

Ideal Transfer Function

$$\omega_0 = \frac{1}{\sqrt{R_1 \cdot R_2 \cdot C_1 \cdot C_2}} \quad \omega_1 = \frac{1}{R_1 \cdot C_1} \quad A = g_m \cdot R_1$$

o -

$$Q = \frac{1}{C_2 \cdot \omega_0 \cdot (R_1 + R_2)}$$

$$V_T := \frac{k \cdot \text{Temp}}{q}$$

$$V_T = 25.899 \text{ mV}$$

$$g_m = \frac{I_C}{V_T}$$

$$\text{Den} = 1 + \frac{1}{\omega_0 \cdot Q} \cdot s + \left(\frac{s}{\omega_0} \right)^2$$

$$A_1(s) = \frac{\left[\left[1 + \left(\frac{s \cdot \omega_1}{\omega_0^2} \right) \right] \cdot \frac{s}{A \cdot \omega_1} + 1 \right]}{\left[\left(1 + \frac{s}{Q \cdot \omega_0} \right) \cdot \frac{s}{A \cdot \omega_1} + \text{Den}(s) \right]}$$

$$A_{\text{max}} = \frac{1}{\left(\frac{1}{Q} \cdot \frac{\omega_0}{\omega_1} + g_m \cdot R_1 \right)}$$

Maximum Attenuation with a fast transistor

$$f_{\text{notch}} = \omega_0 \cdot \sqrt{A}$$

Frequency of the Notch

$$A_{\text{notch}} := \frac{1}{A}$$

Attenuation of the Notch

Equivalent Noise Bandwidth

$$H_{\text{LPF}}(s, G, \omega_0, Q) := \frac{G}{\left(\frac{s}{\omega_0} \right)^2 + \frac{s}{\omega_0} \cdot \frac{1}{Q} + 1} \quad \text{Frequency Response - General LPF Representation (no zeros):}$$

$$\text{NoiseBW}(Q) := \int_0^{10000} \left(|H_{\text{LPF}}(\sqrt{-1} \cdot 2 \cdot \pi \cdot f, 1, 2 \cdot \pi, Q)| \right)^2 df$$

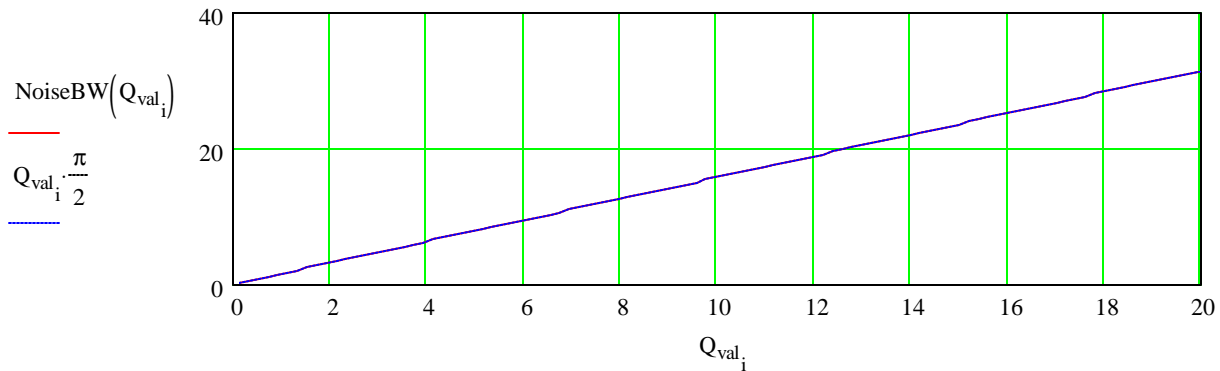
num := 100

Number of Points for Plotting

i := 1..num

Index Vector for Plotting

$$Q_{\text{val}_i} := \frac{i - 1}{\text{num} - 1} \cdot (20 - 0.1) + 0.1$$



Derivation

First Use KCL to solve for the transfer functions for the system

$$\text{KCL @ Node } V_x \quad \frac{V_i - V_x}{R_1} = \frac{V_x - \left(\frac{V_o}{G} + V_n \right)}{R_2} + (V_x - V_o) \cdot C_1 \cdot s$$

$$\text{KCL @ Node } V_o/G \quad \frac{V_x - \left(\frac{V_o}{G} + V_n \right)}{R_2} = \left(\frac{V_o}{G} + V_n \right) \cdot C_2 \cdot s$$

$$\left[\begin{array}{c} 1 \\ \vdots \\ C_2 \cdot (R_2) \end{array} \right] \cdot \left[\begin{array}{c} 1 \\ \vdots \\ 1 \end{array} \right] = \left[\begin{array}{c} 1 \\ \vdots \\ 2 \end{array} \right]$$

$$V_o = \frac{\left[R_1 \cdot C_1 \cdot R_2 \cdot C_2 \cdot s^2 + \left[\frac{C_2}{C_1} \cdot \left(\frac{R_2}{R_1} + 1 \right) + 1 \right] \cdot R_1 \cdot C_1 \cdot s + 1 \right] \cdot G \cdot V_n \dots}{R_1 \cdot C_1 \cdot R_2 \cdot C_2 \cdot s^2 + \left[\frac{C_2}{C_1} \cdot \left(\frac{R_2}{R_1} + 1 \right) + 1 - G \right] \cdot R_1 \cdot C_1 \cdot s + 1} + \frac{G \cdot V_i}{R_1 \cdot C_1 \cdot R_2 \cdot C_2 \cdot s^2 + \left[\frac{C_2}{C_1} \cdot \left(\frac{R_2}{R_1} + 1 \right) + 1 - G \right] \cdot R_1 \cdot C_1 \cdot s + 1}$$

$$\omega_0 = \frac{1}{\sqrt{R_1 \cdot C_1 \cdot R_2 \cdot C_2}}$$

$$Q = \frac{\omega_0}{\frac{1}{R_1 \cdot C_1} + \frac{1}{R_2 \cdot C_1} + \frac{1-G}{R_2 \cdot C_2}} \quad Q = \frac{1}{\left[\frac{C_2}{C_1} \cdot \left(\frac{R_2}{R_1} + 1 \right) + 1 - G \right] \cdot R_1 \cdot C_1 \cdot \omega_0}$$

$$V_{neq}^2 = 4 \cdot k \cdot \text{Temp} \cdot (R_1 + R_2) + V_n^2$$

$$V_{oeq}^2 = V_{neq}^2 \cdot \left(|H_{LPF}(s, G, \omega_0, Q)| \right)^2$$

What is important is the integrated noise.

$$V_{oeq}^2 = V_{neq}^2 \cdot f_0 \cdot \text{NoiseBW} \cdot G^2$$

$$V_{neq}^2 = \frac{V_{oeq}^2}{f_0 \cdot \text{NoiseBW} \cdot G^2}$$



Inputs

$$f_0 := \frac{1.227\text{MHz}}{2}$$

$$f_0 = 613.5 \text{ kHz}$$

Center Frequency

$$Q := 0.7071$$

Desired Q

$$\text{SNR} := 80\text{dB}$$

Minimum Signal to Noise Ratio

$$V_{DD} := 2.7\text{V}$$

Supply Voltage

$$\text{Temp} := 300\text{K}$$

Temperature Range

$$G := 1$$

Gain

$$P_{\text{jammer}} := -30\text{dBm}$$

Power of the Jammer

$$P_{\text{signal}} := -80\text{dBm}$$

Power of the Desired Signal

$$f_{\text{jammer}} := 900\text{kHz}$$

Frequency of Jammer

$$f_{\text{jammer2}} := 1700\text{kHz}$$

Frequency of Second Jammer (for Two-Tone Analysis)

Optional Inputs

$$V_{DSSat} := 0.3\text{V}$$

V_{DSSat} of Op-Amp Input

$$R_{\text{maxdes}} := 400\text{k}\Omega$$

Maximum Desired Resistor

Calculations

$$\omega_0 := 2 \cdot \pi \cdot f_0$$

Effective Noise Bandwidth

$$\text{NoiseBW}(Q) = 1.111$$

$$V_{\text{in swing}} = 2.1 \text{ V}$$

$$V_{\text{in swing}} := V_{DD} - 2 \cdot V_{DSSat}$$

$$V_{\text{out swing}} := V_{\text{in swing}} \cdot \left| H_{LPF}(j \cdot 2 \cdot \pi \cdot f_{\text{jammer}}, G, \omega_0, Q) \right|$$

$$V_{\text{out swing}} = 0.885 \text{ V}$$

$$V_{\text{rms}} := \frac{V_{\text{in swing}}}{2 \cdot \sqrt{2}}$$

$$V_{\text{rms}} = 0.742 \text{ V}$$

$$V_{\text{neq}} := \frac{V_{\text{rms}}}{10^{20}} \cdot \frac{1}{\frac{\text{SNR}}{\sqrt{f_0 \cdot \text{NoiseBW}(Q)}}}$$

$$V_{\text{neq}} = 89.943 \frac{\text{nV}}{\sqrt{\text{Hz}}}$$

$$V_{\text{oeq}} := V_{\text{neq}}^2 \cdot f_0 \cdot \text{NoiseBW}(Q) \cdot G^2$$

$$\sqrt{V_{\text{oeq}}} = 74.246 \mu\text{V}$$

$$V_n := \frac{V_{\text{neq}}}{\sqrt{4}}$$

$$V_n = 44.971 \frac{\text{nV}}{\sqrt{\text{Hz}}}$$

Solving Assumption Number 1: $R_1=R_2$

$$G_{\text{thresh}} := \frac{1}{8 \cdot Q^2} + 1$$

$$G_{\text{thresh}} = 1.25$$

G should be less than or equal to one

$$\text{valid}_r := G \leq \frac{1}{8 \cdot Q^2} + 1$$

$$\text{valid}_r = 1$$

$$R := \frac{V_{\text{neq}}^2 - V_n^2}{2 \cdot 4 \cdot k \cdot \text{Temp}}$$

$$R = 182.794 \text{ k}\Omega$$

$$R := \text{if}(R > 100 \text{ k}\Omega, 100 \text{ k}\Omega, R)$$

$$R = 100 \text{ k}\Omega$$

$$R_{1r} := R$$

$$R_{2r} := R$$

$$C_{2r} := \frac{1 + \sqrt{1 - 8 \cdot Q^2 \cdot (G - 1)}}{4 \cdot Q \cdot \omega_0 \cdot R}$$

$$C_{2r} = 1.834 \text{ pF}$$

Problem with the $R_1=R_2$ architecture. C_1 blows up for large Q's.

$$C_{1r} := \frac{1}{\omega_0^2 \cdot R^2 \cdot C_{2r}}$$

$$C_{1r} = 3.669 \text{ pF}$$

$$\omega_{1r} := \frac{1}{R_{1r} \cdot C_{1r}}$$

$$\frac{\omega_{1r}}{2 \cdot \pi} = 433.814 \text{ kHz}$$

$$\omega_{0r} := \frac{1}{\sqrt{R_{1r} \cdot R_{2r} \cdot C_{1r} \cdot C_{2r}}}$$

$$\frac{\omega_{0r}}{2 \cdot \pi} = 613.5 \text{ kHz}$$

$$Q_r := \frac{\omega_0}{\frac{1}{R_{1r} \cdot C_{1r}} + \frac{1}{R_{2r} \cdot C_{1r}} + \frac{1 - G}{R_{2r} \cdot C_{2r}}}$$

$$Q_r = 0.707$$

The following current should be sized much higher to reduce the distortion of the amplifier.

$$\Delta V_{\text{outswing_}\Delta t} := V_{\text{outswing}} \cdot 2 \cdot \pi \cdot f_{\text{jammer}}$$

Crude Estimate of Load Capacitance

$$C_{\text{Leff}} := C_{1r}$$

$$C_{\text{Leff}} = 3.669 \text{ pF}$$

Estimated Load Capacitance

$$I_{\text{slewr}} := C_{\text{Leff}} \cdot \Delta V_{\text{outswing_}\Delta t}$$

$$I_{\text{slewr}} = 0.018 \text{ mA}$$

Required Current to Slew Output

$$I_{\text{noiser}} := \frac{\left[4 \cdot k \cdot \text{Temp} \cdot \left(\frac{V_T}{2} \right) \right]}{V_n^2}$$

$$I_{\text{noiser}} = 0.106 \mu\text{A}$$

$$I_r := \text{if}(I_{\text{slewr}} > I_{\text{noiser}}, I_{\text{slewr}}, I_{\text{noiser}})$$

$$I_r = 0.018 \text{ mA}$$

$$I_r := \text{if}(I_r < I_{\text{min}}, I_{\text{min}}, I_r)$$

$$g_{\text{mr}} := \frac{I_r}{V_T}$$

$$g_{\text{mr}} = 0.709 \frac{\text{mA}}{\text{V}}$$

$$A_r := g_{mr} \cdot R_{1r}$$

$$A_r = 10.001$$

$$A_{maxr} := \frac{1}{\left(\frac{1}{Q} \cdot \frac{\omega_0}{\omega_{1r}} + g_{mr} \cdot R_{1r} \right)}$$

$$20 \cdot \log(A_{maxr}) = -37.253 \text{ dB} \quad \text{Maximum Attenuation with a fr}$$

$$f_{notchr} := \frac{\omega_0}{2 \cdot \pi} \cdot \sqrt{A_r}$$

$$f_{notchr} = 5.165 \text{ MHz} \quad \text{Frequency of the Notch}$$

$$A_{notchr} := \frac{1}{A_r}$$

$$20 \cdot \log(A_{notchr}) = -37.011 \text{ dB}$$

$$\text{area}_{Rr} := W_{min} \cdot \frac{(R_{1r} + R_{2r}) \cdot W_{min}}{R_{sq}}$$

$$\sqrt{\text{area}_{Rr}} = 10 \mu\text{m}$$

$$\text{cost}_{Rr} := \text{cost_mm2} \cdot \text{area}_{Rr}$$

$$\text{cost}_{Rr} = 1.2 \times 10^{-3} \text{ cent}$$

$$\text{area}_{Cr} := \frac{1}{C_area} \cdot (C_{1r} + C_{2r})$$

$$\sqrt{\text{area}_{Cr}} = 88.666 \mu\text{m}$$

$$\text{cost}_{Cr} := \text{cost_mm2} \cdot \text{area}_{Cr}$$

$$\text{cost}_{Cr} = 0.094 \text{ cent}$$

$$\text{area}_r := \text{area}_{Cr} + \text{area}_{Rr}$$

$$\sqrt{\text{area}_r} = 89.228 \mu\text{m}$$

$$\text{cost}_{power_r} := \text{cost}_{power} \cdot I_r \cdot V_{DD}$$

$$\text{cost}_{power_r} = 0.14 \text{ cent}$$

$$\text{cost}_r := \text{cost}_{Cr} + \text{cost}_{Rr} + \text{cost}_{power_r}$$

$$\text{cost}_r = 0.236 \text{ cent}$$

Solving Assumption Number 2: $C_1=C_2$

$$G_{\text{thresh}} := 1 + \frac{1}{Q} - \frac{1}{2 \cdot Q^2}$$

$$G_{\text{thresh}} = 1.25$$

G should be greater than 2

Problem with C1=C2 architecture: Cannot be used with an emitter follower unless $Q < 1/2$.

$$\text{valid}_c := G \geq 1 + \frac{1}{Q} - \frac{1}{2 \cdot Q^2}$$

$$\text{valid}_c = 0$$

All G's can be used with $Q < \frac{5}{8}$

$$\left[\frac{V_{neq} - V_n}{4 \cdot k \cdot T} \cdot Q + (1 - G) \cdot Q \cdot R_1 \right]^2 + R_1 \cdot \left(R_1 - \frac{V_{neq}^2 - V_n^2}{4 \cdot k \cdot \text{Temp}} \right) = 0$$

$$R_{1c} := \left[\frac{\frac{1}{2} - Q^2 \cdot (1 - G)}{(1 - G)^2 \cdot Q^2 + 1} \right] \cdot \frac{V_{neq}^2 - V_n^2}{4 \cdot k \cdot \text{Temp}} \cdot \left[1 - \sqrt{1 - \frac{1}{\left[\frac{1}{2 \cdot Q} - Q \cdot (1 - G) \right]^2}} \right]$$

$$R_{1c} = 182.794 - 182.79i \text{ k}\Omega$$

$$R_{2c} := \left(\frac{V_{neq}^2 - V_n^2}{4 \cdot k \cdot \text{Temp}} - R_{1c} \right)$$

$$R_{2c} = 182.794 + 182.79i \text{ k}\Omega$$

$$C := \frac{1}{\left[R_{1c} + R_{2c} + (1 - G) \cdot R_{1c} \right] \cdot Q \cdot \omega_0}$$

$$C = 1.004 \text{ pF}$$

$$C_{1c} := C$$

$$C_{2c} := C$$

$$\omega_{0c} := \frac{1}{\sqrt{R_{1c} \cdot R_{2c} \cdot C_{1c} \cdot C_{2c}}}$$

$$\frac{\omega_{0c}}{2 \cdot \pi} = 613.5 \text{ kHz}$$

$$Q_c := \frac{\omega_0}{\frac{1}{R_{1c} \cdot C_{1c}} + \frac{1}{R_{2c} \cdot C_{1c}} + \frac{1 - G}{R_{2c} \cdot C_{2c}}}$$

$$Q_c = 0.707$$

$$C_{1c} = C$$

$$C_{2c} = C$$

$$C_{Leff} := C_{1c}$$

$$I_{slewc} := C_{Leff} \cdot \Delta V_{outswing} \cdot \Delta t$$

$$I_{noisec} := \frac{\left[4 \cdot k \cdot \text{Temp} \cdot \left(\frac{V_T}{2} \right) \right]}{V_n^2}$$

$$I_c := \text{if}(I_{slewc} > I_{noisec}, I_{slewc}, I_{noisec})$$

$$I_c := \text{if}(I_c < I_{min}, I_{min}, I_c)$$

$$\omega_{1c} := \frac{1}{R_{1c} \cdot C_{1c}}$$

$$g_{mc} := \frac{I_c}{V_T}$$

$$A_c := g_{mc} \cdot R_{1c}$$

$$A_{maxc} := \frac{1}{\left(\frac{1}{Q} \cdot \frac{\omega_0}{\omega_{1c}} + g_{mc} \cdot R_{1c} \right)}$$

$$f_{notchc} := \frac{\omega_0}{2 \cdot \pi} \cdot \sqrt{A_c}$$

$$A_{notchc} := \frac{1}{A_c}$$

$$\text{area}_{Rc} := W_{min} \cdot \frac{(R_{1c} + R_{2c}) \cdot W_{min}}{R_{sq}}$$

$$\text{cost}_{Rc} := \text{cost}_{mm2} \cdot \text{area}_{Rc}$$

$$\text{area}_{Cc} := \frac{1}{C_{area}} \cdot (C_{1c} + C_{2c})$$

$$\text{cost}_{Cc} := \text{cost}_{mm2} \cdot \text{area}_{Cc}$$

$$\text{area}_c := \text{area}_{Cc} + \text{area}_{Rc}$$

$$\text{cost}_{powerc} := \text{cost}_{power} \cdot I_c \cdot V_{DD}$$

$$\text{cost}_c := \text{cost}_{Cc} + \text{cost}_{Rc} + \text{cost}_{powerc}$$

Solving Assumption Number 3: $R_1 C_1 = R_2 C_2 = \tau$.

$$G_{thresh} := 2 - \frac{1}{Q}$$

$$\text{valid}_t := G > G_{thresh}$$

$$R_{2t} := \frac{V_{neq}^2 - V_n^2}{4 \cdot k \cdot \text{Temp} \cdot \left(\frac{1}{Q} + G - 1 \right)}$$

$$R_{1t} := \left(\frac{1}{Q} + G - 2 \right) R_{2t}$$

$$C_{1t} := \frac{1}{R_{1t} \cdot \omega_0}$$

$$C_{2t} := \frac{1}{R_{2t} \cdot \omega_0}$$

$$\omega_{0t} := \frac{1}{\sqrt{R_{1t} \cdot R_{2t} \cdot C_{1t} \cdot C_{2t}}}$$

$$C_{Leff} = 1.004 \text{ pF}$$

$$I_{slewc} = 5.022 \mu\text{A}$$

$$I_{noisec} = 0.106 \mu\text{A}$$

$$I_c = 5.022 \mu\text{A}$$

$$\frac{\omega_{1c}}{2 \cdot \pi} = 433.814 + 433.806i \text{ kHz}$$

$$g_{mc} = 0.386 \frac{\text{mA}}{\text{V}}$$

$$A_c = 70.58 - 70.578i$$

$$20 \cdot \log(|A_{maxc}|) = -40.106 \text{ dB}$$

$$f_{notchc} = 5.663 - 2.346i \text{ MHz}$$

$$20 \cdot \log(A_{notchc}) = -39.984 + 6.822i \text{ dB}$$

$$\sqrt{\text{area}_{Rc}} = 13.52 \mu\text{m}$$

$$\text{cost}_{Rc} = 2.194 \times 10^{-3} \text{ cent}$$

$$\sqrt{\text{area}_{Cc}} = 53.547 \mu\text{m}$$

$$\text{cost}_{Cc} = 0.034 \text{ cent}$$

$$\sqrt{\text{area}_c} = 55.227 \mu\text{m}$$

$$\text{cost}_{powerc} = 0.076 \text{ cent}$$

$$\text{cost}_c = 0.113 \text{ cent}$$

Required Current to Slew Output

Maximum Attenua
with a fast transist

Frequency of the

Depth of Notch

$$\sqrt{\omega_{0t}^2 - \omega_{nt}^2}$$

$$2 \cdot \pi$$

$$Q_t := \frac{\omega_0}{\frac{1}{R_{1t} \cdot C_{1t}} + \frac{1}{R_{2t} \cdot C_{1t}} + \frac{1-G}{R_{2t} \cdot C_{2t}}}$$

$$Q_t = 0.707$$

Solving Assumption #4: $C_1 = C_{\max}$ or C_{\min}

First solve for maximum Capacitance by setting the cost of the internal capacitor to that of an external capacitor. This upper limit is set when extra pins are available to put a capacitor off-chip.

$$C_{\max} := \frac{C_{\text{area}}}{\text{cost_mm}^2} \cdot \text{costCext}$$

$$C_{\max} = 183.96 \text{ pF}$$

$$\text{Area}_{C_{\max}} := \frac{C_{\max}}{C_{\text{area}}}$$

$$\sqrt{\text{Area}_{C_{\max}}} = 512.64 \mu\text{m}$$

If the pins are not available to put the capacitor off-chip, the maximum capacitor size must be re-evaluated using marketing estimates for the amount the chip can sell for, and yields given the larger chip size, and package limits on the die size.

$$C_{\max} := 100 \text{ pF}$$

Maximum Desired On-Chip Capacitance

$$\text{Area}_{C_{\max}} := \frac{C_{\max}}{C_{\text{area}}}$$

$$\sqrt{\text{Area}_{C_{\max}}} = 512.64 \mu\text{m}$$

Given the center frequency, maximum capacitor size, and desired SNDR the needed capacitance is given by the following equation to prevent a complex resistor sizing.

$$C_{\text{maxm}} := \frac{Q}{\omega_0} \cdot \frac{4 \cdot k \cdot \text{Temp} \cdot 4}{V_{\text{neq}}^2 - V_n^2}$$

$$C_{\text{maxm}} = 2.007 \text{ pF}$$

$$\text{Area}_{C_{\text{maxm}}} := \frac{C_{\text{maxm}}}{C_{\text{area}}}$$

$$\sqrt{\text{Area}_{C_{\text{maxm}}}} = 53.546 \mu\text{m}$$

Now Solve for Variables

$$C_{1m} := C_{\text{maxm}} \cdot 1.05$$

$$C_{1m} = 2.107 \text{ pF}$$

$$\text{valid}_m := Q < \omega_0 \cdot C_{1m} \cdot \left(\frac{V_{\text{neq}}^2 - V_n^2}{4 \cdot k \cdot \text{Temp} \cdot 4} \right)$$

$$\text{valid}_m = 1$$

$$R_{2m} := \frac{V_{\text{neq}}^2 - V_n^2}{4 \cdot k \cdot \text{Temp} \cdot 2} \cdot \left[1 + \sqrt{1 - \frac{(4 \cdot k \cdot \text{Temp}) \cdot 4}{(V_{\text{neq}}^2 - V_n^2)} \cdot \frac{Q}{\omega_0 \cdot C_{1m}}} \right]$$

$$R_{2m} = 222.683 \text{ k}\Omega$$

$$R_{1m} := \frac{V_{\text{neq}}^2 - V_n^2}{4 \cdot k \cdot \text{Temp}} - R_{2m}$$

$$R_{1m} = 142.905 \text{ k}\Omega$$

$$C_{2m} := \frac{1}{\omega_0^2 \cdot R_{1m} \cdot R_{2m} \cdot C_{1m}}$$

$$C_{2m} = 1.004 \text{ pF}$$

$$\omega_{0m} := \frac{1}{\sqrt{R_{1m} \cdot R_{2m} \cdot C_{1m} \cdot C_{2m}}}$$

$$\frac{\omega_{0m}}{2 \cdot \pi} = 613.5 \text{ kHz}$$

$$Q_m := \frac{\omega_0}{\frac{1}{R_{1m} \cdot C_{1m}} + \frac{1}{R_{2m} \cdot C_{1m}} + \frac{1-G}{R_{2m} \cdot C_{2m}}}$$

$$Q_m = 0.707$$

$$Q_{\text{maxm}} := \omega_0 \cdot C_{1m} \cdot \left(\frac{V_{\text{neq}}^2 - V_n^2}{4 \cdot k \cdot \text{Temp} \cdot 4} \right)$$

$$Q_{\text{maxm}} = 0.742$$

$$C_{\text{Leff}} := C_{1m}$$

$$C_{\text{Leff}} = 2.107 \text{ pF}$$

Estimated Load Capacitance

$$I_{\text{slewm}} := C_{\text{Leff}} \cdot \Delta V_{\text{outswing}} \cdot \Delta t$$

$$I_{\text{slewm}} = 10.546 \mu\text{A}$$

Required Current
to Slew Output

$$\left[4 \cdot k \cdot \text{Temp} \cdot \left(\frac{V_T}{\gamma} \right) \right]$$

$$I_{\text{noisem}} := \frac{L \cdot \omega \cdot \mu}{V_n^2}$$

$$I_{\text{noisem}} = 0.106 \mu\text{A}$$

$$I_m := \text{if}(I_{\text{slewm}} > I_{\text{noisem}}, I_{\text{slewm}}, I_{\text{noisem}})$$

$$I_m = 10.546 \mu\text{A}$$

$$I_m := \text{if}(I_m < I_{\text{min}}, I_{\text{min}}, I_m)$$

$$\omega_{1m} := \frac{1}{R_{1m} \cdot C_{1m}}$$

$$\frac{\omega_{1m}}{2 \cdot \pi} = 528.48 \text{ kHz}$$

$$g_{mm} := \frac{I_m}{V_T}$$

$$g_{mm} = 0.407 \frac{\text{mA}}{\text{V}}$$

$$A_m := g_{mm} \cdot R_{1m}$$

$$A_m = 58.189$$

$$A_{\text{maxm}} := \frac{1}{\left(\frac{1}{Q} \cdot \frac{\omega_0}{\omega_{1m}} + g_{mm} \cdot R_{1m} \right)}$$

$$20 \cdot \log(|A_{\text{maxm}}|) = -35.538 \text{ dB} \quad \text{Maximum Attenuation with a fast transistor}$$

$$f_{\text{notchm}} := \frac{\omega_0}{2 \cdot \pi} \cdot \sqrt{A_m}$$

$$f_{\text{notchm}} = 4.68 \text{ MHz} \quad \text{Frequency of the Notch}$$

$$A_{\text{notchm}} := \frac{1}{A_m}$$

$$20 \cdot \log(A_{\text{notchm}}) = -35.297 \text{ dB} \quad \text{Depth of Notch}$$

$$\text{area}_{Rm} := W_{\text{min}} \cdot \frac{(R_{1m} + R_{2m}) \cdot W_{\text{min}}}{R_{\text{sq}}}$$

$$\sqrt{\text{area}_{Rm}} = 13.52 \mu\text{m}$$

$$\text{cost}_{Rm} := \text{cost}_{\text{mm}2} \cdot \text{area}_{Rm}$$

$$\text{cost}_{Rm} = 2.194 \times 10^{-3} \text{ cent}$$

$$\text{area}_{Cm} := \frac{1}{C_{\text{area}}} \cdot (C_{1m} + C_{2m})$$

$$\sqrt{\text{area}_{Cm}} = 66.665 \mu\text{m}$$

$$\text{cost}_{Cm} := \text{cost}_{\text{mm}2} \cdot \text{area}_{Cm}$$

$$\text{cost}_{Cm} = 0.053 \text{ cent}$$

$$\text{area}_m := \text{area}_{Cm} + \text{area}_{Rm}$$

$$\sqrt{\text{area}_m} = 68.022 \mu\text{m}$$

$$\text{cost}_{\text{powerm}} := \text{cost}_{\text{power}} \cdot I_m \cdot V_{DD}$$

$$\text{cost}_{\text{powerm}} = 0.081 \text{ cent}$$

$$\text{cost}_m := \text{cost}_{Cm} + \text{cost}_{Rm} + \text{cost}_{\text{powerm}}$$

$$\text{cost}_m = 0.136 \text{ cent}$$

[Solving Assumption #4.5: Other Solution to Quadratic of 4.](#)

$$C_{1m2} := C_{\text{maxm}} \cdot 1.05$$

$$C_{1m2} = 2.107 \text{ pF}$$

$$\text{valid}_{m2} := Q < \omega_0 \cdot C_{1m2} \cdot \left(\frac{V_{\text{neq}}^2 - V_n^2}{4 \cdot k \cdot \text{Temp} \cdot 4} \right)$$

$$\text{valid}_{m2} = 1$$

$$R_{2m2} := \frac{V_{\text{neq}}^2 - V_n^2}{4 \cdot k \cdot \text{Temp} \cdot 2} \cdot \left[1 - \sqrt{1 - \frac{(4 \cdot k \cdot \text{Temp}) \cdot 4}{(V_{\text{neq}}^2 - V_n^2)} \cdot \frac{Q}{\omega_0 \cdot C_{1m2}}} \right]$$

$$R_{2m2} = 142.905 \text{ k}\Omega$$

$$R_{1m2} := \frac{V_{\text{neq}}^2 - V_n^2}{4 \cdot k \cdot \text{Temp}} - R_{2m2}$$

$$R_{1m2} = 222.683 \text{ k}\Omega$$

$$C_{2m2} := \frac{1}{\omega_0^2 \cdot R_{1m2} \cdot R_{2m2} \cdot C_{1m2}}$$

$$C_{2m2} = 1.004 \text{ pF}$$

$$\omega_{0m2} := \frac{1}{\sqrt{R_{1m2} \cdot R_{2m2} \cdot C_{1m2} \cdot C_{2m2}}}$$

$$\frac{\omega_{0m2}}{2 \cdot \pi} = 613.5 \text{ kHz}$$

$$Q_{m2} := \frac{\omega_0}{\frac{1}{R_{1m2} \cdot C_{1m2}} + \frac{1}{R_{2m2} \cdot C_{1m2}} + \frac{1 - G}{R_{2m2} \cdot C_{2m2}}}$$

$$Q_{m2} = 0.707$$

$$\left(\frac{V_{\text{neq}}^2 - V_n^2}{4 \cdot k \cdot \text{Temp} \cdot 4} \right)$$

$$Q_{\max 2} := \omega_0 \cdot C_{1m2} \cdot \left(\frac{1}{4 \cdot k \cdot \text{Temp} \cdot 4} \right)$$

$$C_{\text{Leff}} := C_{1m2}$$

$$I_{\text{slewm}2} := C_{\text{Leff}} \cdot \Delta V_{\text{outswing}} \cdot \Delta t$$

$$I_{\text{noisem}2} := \frac{\left[4 \cdot k \cdot \text{Temp} \cdot \left(\frac{V_T}{2} \right) \right]}{V_n^2}$$

$$I_{m2} := \text{if}(I_{\text{slewm}2} > I_{\text{noisem}2}, I_{\text{slewm}2}, I_{\text{noisem}2})$$

$$I_{m2} := \text{if}(I_{m2} < I_{\min}, I_{\min}, I_{m2})$$

$$\omega_{1m2} := \frac{1}{R_{1m2} \cdot C_{1m2}}$$

$$g_{mm2} := \frac{I_{m2}}{V_T}$$

$$A_{m2} := g_{mm2} \cdot R_{1m2}$$

$$A_{\max 2} := \frac{1}{\left(\frac{1}{Q} \cdot \frac{\omega_0}{\omega_{1m2}} + g_{mm2} \cdot R_{1m2} \right)}$$

$$f_{\text{notchm}2} := \frac{\omega_0}{2 \cdot \pi} \cdot 2 \cdot \sqrt{A_m}$$

$$A_{\text{notchm}2} := \frac{1}{A_{m2}}$$

$$\text{area}_{Rm2} := W_{\min} \cdot \frac{(R_{1m2} + R_{2m2}) \cdot W_{\min}}{R_{sq}}$$

$$\text{cost}_{Rm2} := \text{cost}_{mm2} \cdot \text{area}_{Rm2}$$

$$\text{area}_{Cm2} := \frac{1}{C_{\text{area}}} \cdot (C_{1m2} + C_{2m2})$$

$$\text{cost}_{Cm2} := \text{cost}_{mm2} \cdot \text{area}_{Cm2}$$

$$\text{area}_{m2} := \text{area}_{Cm2} + \text{area}_{Rm2}$$

$$\text{cost}_{\text{powerm}2} := \text{cost}_{\text{power}} \cdot I_{m2} \cdot V_{DD}$$

$$\text{cost}_{m2} := \text{cost}_{Cm2} + \text{cost}_{Rm2} + \text{cost}_{\text{powerm}2}$$

Solving Assumption #5: $R_1 = R_{\max}$

$$R_{\max} := \frac{\text{cost}_{R\text{ext}} \cdot R_{sq}}{\text{cost}_{mm2} \cdot W_{\min}^2}$$

$$R_{\max} := \text{if}(R_{\max} < R_{\max\text{des}}, R_{\max}, R_{\max\text{des}})$$

$$\text{Area}_{R\max} := W_{\min} \cdot \frac{R_{\max} \cdot W_{\min}}{R_{sq}}$$

$$R_{1n} := R_{\max}$$

$$R_{2n} := \frac{V_{\text{neq}}^2 - V_n^2}{4 \cdot k \cdot \text{Temp}} - R_{1n}$$

$$C_{1n} := \left[\left(\frac{1}{R_{1n}} + \frac{1}{R_{2n}} \right) \cdot \frac{Q}{\omega_0} \right] \text{ if } G = 1$$

$$Q_{\max 2} = 0.142$$

$$C_{\text{Leff}} = 2.107 \text{ pF}$$

$$I_{\text{slewm}2} = 10.546 \mu\text{A}$$

$$I_{\text{noisem}2} = 0.106 \mu\text{A}$$

$$I_{m2} = 10.546 \mu\text{A}$$

$$\frac{\omega_{1m2}}{2 \cdot \pi} = 339.148 \text{ kHz}$$

$$g_{mm2} = 0.407 \frac{\text{mA}}{\text{V}}$$

$$A_{m2} = 90.673$$

$$20 \cdot \log(|A_{\max 2}|) = -39.391 \text{ dB}$$

Maximum Attenuation with a fast transistor

$$f_{\text{notchm}2} = 9.36 \text{ MHz}$$

Frequency of the Notch

$$20 \cdot \log(A_{\text{notchm}2}) = -39.15 \text{ dB}$$

Depth of Notch

$$\sqrt{\text{area}_{Rm2}} = 13.52 \mu\text{m}$$

$$\text{cost}_{Rm2} = 2.194 \times 10^{-3} \text{ cent}$$

$$\sqrt{\text{area}_{Cm2}} = 66.665 \mu\text{m}$$

$$\text{cost}_{Cm2} = 0.053 \text{ cent}$$

$$\sqrt{\text{area}_{m2}} = 68.022 \mu\text{m}$$

$$\text{cost}_{\text{powerm}2} = 0.081 \text{ cent}$$

$$\text{cost}_{m2} = 0.136 \text{ cent}$$

$$R_{\max} = 442.267 \text{ M}\Omega$$

$$R_{\max} = 400 \text{ k}\Omega$$

$$\sqrt{\text{Area}_{R\max}} = 14.142 \mu\text{m}$$

$$R_{1n} = 400 \text{ k}\Omega$$

$$R_{2n} = -34.413 \text{ k}\Omega$$

$$\left\| \frac{\omega_0}{2 \cdot Q} \cdot \frac{1}{(1 - G) \cdot \omega_0^2 \cdot R_{1n}} \cdot \left[1 - \sqrt{1 - \left(1 + \frac{R_{1n}}{R_{2n}} \right) \cdot 4 \cdot (1 - G) \cdot Q^2} \right] \right\| \text{ if } G \neq 1$$

$$G_{\text{thresh}} := 1 - \frac{1}{4 \cdot Q^2 \cdot \left(1 + \frac{R_{1n}}{R_{2n}} \right)}$$

$$\text{valid}_n := (G > G_{\text{thresh}}) \cdot (R_{2n} > 0)$$

$$C_{2n} := \frac{1}{\omega_0^2 \cdot R_{1n} \cdot R_{2n} \cdot C_{1n}}$$

$$\omega_{0n} := \frac{1}{\sqrt{R_{1n} \cdot R_{2n} \cdot C_{1n} \cdot C_{2n}}}$$

$$Q_n := \frac{\omega_0}{\frac{1}{R_{1n} \cdot C_{1n}} + \frac{1}{R_{2n} \cdot C_{1n}} + \frac{1 - G}{R_{2n} \cdot C_{2n}}}$$

$$C_{\text{Leff}} := C_{1n}$$

$$I_{\text{slewn}} := C_{\text{Leff}} \cdot \Delta V_{\text{outswing}} \cdot \Delta t$$

$$I_{\text{noisen}} := \frac{\left[4 \cdot k \cdot \text{Temp} \cdot \left(\frac{V_T}{2} \right) \right]}{V_n^2}$$

$$I_n := \text{if}(I_{\text{slewn}} > I_{\text{noisen}}, I_{\text{slewn}}, I_{\text{noisen}})$$

$$I_n := \text{if}(I_n < I_{\text{min}}, I_{\text{min}}, I_n)$$

$$\omega_{1n} := \frac{1}{R_{1n} \cdot C_{1n}}$$

$$g_{mn} := \frac{I_n}{V_T}$$

$$A_n := g_{mn} \cdot R_{1n}$$

$$A_{\text{maxn}} := \frac{1}{\left(\frac{1}{Q} \cdot \frac{\omega_0}{\omega_{1n}} + g_{mn} \cdot R_{1n} \right)}$$

$$f_{\text{notchn}} := \frac{\omega_0}{2 \cdot \pi} \cdot \sqrt{A_n}$$

$$A_{\text{notchn}} := \frac{1}{A_n}$$

$$\text{area}_{Rn} := W_{\text{min}} \cdot \frac{(R_{1n} + R_{2n}) \cdot W_{\text{min}}}{R_{\text{sq}}}$$

$$\text{cost}_{Rn} := \text{cost_mm2} \cdot \text{area}_{Rn}$$

$$\text{area}_{Cn} := \frac{1}{C_{\text{area}}} \cdot (C_{1n} + C_{2n})$$

$$\text{cost}_{Cn} := \text{cost_mm2} \cdot \text{area}_{Cn}$$

$$\text{area}_n := \text{area}_{Cn} + \text{area}_{Rn}$$

$$\text{cost}_{\text{power}_n} := \text{cost}_{\text{power}} \cdot I_n \cdot V_{DD}$$

$$C_{1n} = -4.872 \text{ pF}$$

$$G_{\text{thresh}} = 1.047$$

$$\text{valid}_n = 0$$

$$C_{2n} = 1.004 \text{ pF}$$

$$\frac{\omega_{0n}}{2 \cdot \pi} = 613.5 \text{ kHz}$$

$$Q_n = 0.707$$

$$C_{\text{Leff}} = -4.872 \text{ pF}$$

$$I_{\text{slewn}} = -24.38 \text{ } \mu\text{A}$$

$$I_{\text{noisen}} = 0.106 \text{ } \mu\text{A}$$

$$I_n = 10 \text{ } \mu\text{A}$$

$$\frac{\omega_{1n}}{2 \cdot \pi} = -81.67 \text{ kHz}$$

$$g_{mn} = 0.386 \frac{\text{mA}}{\text{V}}$$

$$A_n = 154.447$$

$$20 \cdot \log(|A_{\text{maxn}}|) = -43.157 \text{ dB}$$

$$f_{\text{notchn}} = 7.624 \text{ MHz}$$

$$20 \cdot \log(A_{\text{notchn}}) = -43.776 \text{ dB}$$

$$\sqrt{\text{area}_{Rn}} = 13.52 \text{ } \mu\text{m}$$

$$\text{cost}_{Rn} = 2.194 \times 10^{-3} \text{ cent}$$

$$\sqrt{\text{area}_{Cn}} = 74.339i \text{ } \mu\text{m}$$

$$\text{cost}_{Cn} = -0.066 \text{ cent}$$

$$\sqrt{\text{area}_n} = 73.099i \text{ } \mu\text{m}$$

$$\text{cost}_{\text{power}_n} = 0.076 \text{ cent}$$

G must be greater than 0.773 for C1 to be real

Estimated Load Capacitance

Required Current to Slew Output

Maximum Attenuation with a fast transistor

Frequency of the Notch

Depth of Notch

$$\text{cost}_n := \text{cost}_{C_n} + \text{cost}_{R_n} + \text{cost}_{\text{power}_n}$$

$$\text{cost}_n = 0.012 \text{ cent}$$

Solving Assumption #6: Minimize Area

This doesn't work for $G < 1$. It spits out unrealistically sized values for G 's close to one. Lets set a threshold of a G of about 1.3. In general the area is dominated by the capacitor

$$R_{1g} := 0.6 \text{ M}\Omega$$

Guess at R_1

$$\text{valid}_a := G > 1.3$$

$$\text{valid}_a = 0$$

$$R_{1a} := \text{root} \left[\left(\frac{1}{R_{1g}} + \frac{1}{R_{1g} - \frac{V_{\text{neq}}^2 - V_n^2}{4 \cdot k \cdot \text{Temp}}} \right) \cdot \frac{1}{\frac{W_{\text{min}}^2 \cdot C_{\text{area}}}{R_{\text{sq}}} \cdot R_{1g}} + (1 - G) \cdot R_{1g}^2 \cdot \frac{W_{\text{min}}^2 \cdot C_{\text{area}}}{R_{\text{sq}}} \cdot \omega_0^2 - \frac{Q}{\omega_0}, R_{1g} \right]$$

$$R_{1a} = 2.627 \times 10^8 \text{ k}\Omega$$

$$R_{2a} := R_{1a} - \frac{V_{\text{neq}}^2 - V_n^2}{4 \cdot k \cdot \text{Temp}}$$

$$R_{2a} = 2.627 \times 10^8 \text{ k}\Omega$$

$$C_{1a} := \frac{W_{\text{min}}^2 \cdot C_{\text{area}}}{R_{\text{sq}}} \cdot R_{1a}$$

$$C_{1a} = 9.194 \times 10^4 \text{ pF}$$

$$C_{2a} := \frac{1}{\left(R_{1a}^2 \cdot \frac{W_{\text{min}}^2 \cdot C_{\text{area}}}{R_{\text{sq}}} \cdot \omega_0^2 \right)} \cdot R_{2a}$$

$$C_{2a} = 0 \text{ pF}$$

Outputs

Implementation, where $C_1=C_2=C$

$$\text{valid}_c = 0$$

$$R_{1c} = 182.794 - 182.79i \text{ k}\Omega$$

$$R_{2c} = 182.794 + 182.79i \text{ k}\Omega$$

$$C_{1c} = 1.004 \text{ pF}$$

$$C_{2c} = 1.004 \text{ pF}$$

$$I_c = 10 \mu\text{A}$$

$$20 \cdot \log(|A_{\text{maxc}}|) = -40.106 \text{ dB}$$

$$f_{\text{notchc}} = 5.663 - 2.346i \text{ MHz}$$

$$20 \cdot \log(A_{\text{notchc}}) = -39.984 + 6.822i \text{ dB}$$

$$\text{cost}_c = 0.113 \text{ cent}$$

Are these Coefficients Valid? 0=no, 1=yes

Resistor 1 Value

Resistor 2 Value

Capacitor Value

Capacitor Value

Required Current to Slew Output

Maximum Attenuation with a fast transistor

Frequency of the Notch

Depth of Notch

Cost of c method

Implementation, where $R_1=R_2=R$

$$\text{valid}_r = 1$$

$$R_{1r} = 100 \text{ k}\Omega$$

$$R_{2r} = 100 \text{ k}\Omega$$

$$C_{1r} = 3.669 \text{ pF}$$

$$C_{2r} = 1.834 \text{ pF}$$

$$I_r = 18.359 \mu\text{A}$$

$$20 \cdot \log(|A_{\text{maxr}}|) = -37.253 \text{ dB}$$

$$f_{\text{notchr}} = 5.165 \text{ MHz}$$

$$20 \cdot \log(A_{\text{notchr}}) = -37.011 \text{ dB}$$

$$\text{cost}_r = 0.236 \text{ cent}$$

Are these Coefficients Valid? 0=no, 1=yes

Resistor 1 Value

Resistor 2 Value

Capacitor 1 Value

Capacitor 2 Value

Required Current to Slew Output

Maximum Attenuation with a fast transistor

Frequency of the Notch

Depth of Notch

Cost of r method

Implementation, where $C_1=C_{max}$

$$\text{valid}_m = 1$$

$$R_{1m} = 142.905 \text{ k}\Omega$$

$$R_{2m} = 222.683 \text{ k}\Omega$$

$$C_{1m} = 2.107 \text{ pF}$$

$$C_{2m} = 1.004 \text{ pF}$$

$$I_m = 10.546 \mu\text{A}$$

$$20 \cdot \log(|A_{\text{maxm}}|) = -35.538 \text{ dB}$$

$$f_{\text{notchm}} = 4.68 \text{ MHz}$$

$$20 \cdot \log(A_{\text{notchm}}) = -35.297 \text{ dB}$$

$$\text{cost}_m = 0.136 \text{ cent}$$

2nd Implementation, where $C_1=C_{max}$

$$\text{valid}_{m2} = 1$$

$$R_{1m2} = 222.683 \text{ k}\Omega$$

$$R_{2m2} = 142.905 \text{ k}\Omega$$

$$C_{1m2} = 2.107 \text{ pF}$$

$$C_{2m2} = 1.004 \text{ pF}$$

$$I_{m2} = 10.546 \mu\text{A}$$

$$20 \cdot \log(|A_{\text{maxm2}}|) = -39.391 \text{ dB}$$

$$f_{\text{notchm2}} = 9.36 \text{ MHz}$$

$$20 \cdot \log(A_{\text{notchm2}}) = -39.15 \text{ dB}$$

$$\text{cost}_{m2} = 0.136 \text{ cent}$$

Implementation, where $R_1=R_{max}$

$$\text{valid}_n = 0$$

$$R_{1n} = 400 \text{ k}\Omega$$

$$R_{2n} = -34.413 \text{ k}\Omega$$

$$C_{1n} = -4.872 \text{ pF}$$

$$C_{2n} = 1.004 \text{ pF}$$

$$I_n = 10 \mu\text{A}$$

$$20 \cdot \log(|A_{\text{maxn}}|) = -43.157 \text{ dB}$$

$$f_{\text{notchn}} = 7.624 \text{ MHz}$$

$$20 \cdot \log(A_{\text{notchn}}) = -43.776 \text{ dB}$$

$$\text{cost}_n = 0.012 \text{ cent}$$

$$\text{architecture} := \begin{cases} 1 & \text{if } \text{valid}_r \\ 2 & \text{if } \text{valid}_c \\ 3 & \text{if } \text{valid}_m \\ \text{error}(\text{"none are valid"}) & \text{if } (1 - \text{valid}_m) \cdot (1 - \text{valid}_c) \cdot (1 - \text{valid}_r) \end{cases}$$

$$\text{architecture} = 3$$

$$R_1 := \begin{cases} R_{1r} & \text{if } \text{valid}_r \\ R_{1c} & \text{if } \text{valid}_c \end{cases}$$

Are these Coefficients Valid? 0=no, 1=yes

Resistor 1 Value

Resistor 2 Value

Capacitor 1 Value

Capacitor 2 Value

Required Current to Slew Output

Maximum Attenuation with a fast transistor

Frequency of the Notch

Depth of Notch

Cost of m method

Are these Coefficients Valid? 0=no, 1=yes

Resistor 1 Value

Resistor 2 Value

Capacitor 1 Value

Capacitor 2 Value

Required Current to Slew Output

Maximum Attenuation with a fast transistor

Frequency of the Notch

Depth of Notch

Cost of m2 method

Are these Coefficients Valid? 0=no, 1=yes

Resistor 1 Value

Resistor 2 Value

Capacitor 1 Value

Capacitor 2 Value

Required Current to Slew Output

Maximum Attenuation with a fast transistor

Frequency of the Notch

Depth of Notch

Cost of n method

$$R_{1m} \text{ if valid}_m$$

$$\text{error("none are valid")} \cdot \Omega \text{ if } (1 - \text{valid}_m) \cdot (1 - \text{valid}_c) \cdot (1 - \text{valid}_r)$$

$$R_1 = 142.905 \text{ k}\Omega$$

$$R_2 := \begin{cases} R_{2r} & \text{if valid}_r \\ R_{2c} & \text{if valid}_c \\ R_{2m} & \text{if valid}_m \\ \text{error("none are valid")} \cdot \Omega & \text{if } (1 - \text{valid}_m) \cdot (1 - \text{valid}_c) \cdot (1 - \text{valid}_r) \end{cases}$$

$$R_2 = 222.683 \text{ k}\Omega$$

$$C_1 := \begin{cases} C_{1r} & \text{if valid}_r \\ C_{1c} & \text{if valid}_c \\ C_{1m} & \text{if valid}_m \\ \text{error("none are valid")} \cdot F & \text{if } (1 - \text{valid}_m) \cdot (1 - \text{valid}_c) \cdot (1 - \text{valid}_r) \end{cases}$$

$$C_1 = 2.107 \text{ pF}$$

$$C_2 := \begin{cases} C_{2r} & \text{if valid}_r \\ C_{2c} & \text{if valid}_c \\ C_{2m} & \text{if valid}_m \\ \text{error("none are valid")} \cdot F & \text{if } (1 - \text{valid}_m) \cdot (1 - \text{valid}_c) \cdot (1 - \text{valid}_r) \end{cases}$$

$$C_2 = 1.004 \text{ pF}$$

$$I := \begin{cases} I_r & \text{if valid}_r \\ I_c & \text{if valid}_c \\ I_m & \text{if valid}_m \\ \text{error("none are valid")} \cdot A & \text{if } (1 - \text{valid}_m) \cdot (1 - \text{valid}_c) \cdot (1 - \text{valid}_r) \end{cases}$$

$$I = 0.011 \text{ mA}$$

$$A_{\max} := \begin{cases} A_{\max r} & \text{if valid}_r \\ A_{\max c} & \text{if valid}_c \\ A_{\max m} & \text{if valid}_m \\ \text{error("none are valid")} & \text{if } (1 - \text{valid}_m) \cdot (1 - \text{valid}_c) \cdot (1 - \text{valid}_r) \end{cases}$$

$$20 \cdot \log(|A_{\max}|) = -35.538 \text{ dB}$$

$$g_m := \frac{I}{V_T}$$

$$g_m = 0.407 \frac{\text{mA}}{\text{V}}$$

$$A := g_m \cdot R_1$$

$$A = 58.189$$

$$f_{\text{notch}} := \frac{\omega_0}{(2 \cdot \omega_1)} \cdot \left[-1 + \sqrt{1 + 4 \cdot \left(\frac{\omega_1}{\omega_0} \right)^2 \cdot A} \right] \cdot \frac{\omega_0}{2 \cdot \pi}$$

$$f_{\text{notch}} = \blacksquare \text{ MHz}$$

Useful bandwidth

$$f_{\text{notchguess}} := \sqrt{A} \cdot \frac{\omega_0}{2 \cdot \pi}$$

$$f_{\text{notchguess}} = 4.68 \text{ MHz}$$

$$A_{\text{notch}} := 20 \cdot \log(|A_1(A, \omega_1, j \cdot 2 \cdot \pi \cdot f_{\text{notch}})|)$$

$$A_{\text{notch}} = \blacksquare \text{ dB}$$

$$A_{\text{jammer}} := 20 \cdot \log(|A_1(A, \omega_1, j \cdot 2 \cdot \pi \cdot f_{\text{jammer}})|)$$

$$A_{\text{jammer}} = \blacksquare \text{ dB}$$

$$20 \cdot \log(|A_1(A, \omega_1, j \cdot 2 \cdot \pi \cdot f_{\text{notchguess}})|) = \blacksquare \text{ dB}$$

$$f_{\text{start}} := \frac{f_0}{10}$$

Starting Frequency for Plotting

$$f_{\text{stop}} := f_0 \cdot 100$$

Stopping Frequency for Plotting

$$f_i := \left(\frac{f_{\text{stop}}}{f_{\text{start}}} \right)^{\frac{i}{\text{num}}} \cdot f_{\text{start}}$$

$$\omega_i := 2 \cdot \pi \cdot f_i$$

$$s_i := j \cdot \omega_i$$

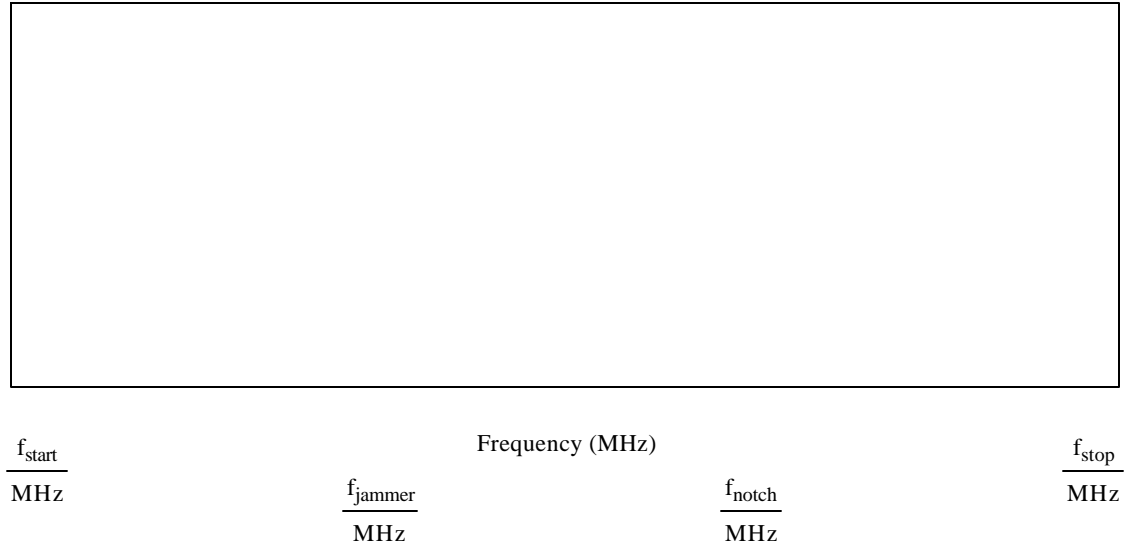
Filter Response vs. Frequency

$$744 \times 10^{-4}$$

$$A_{\text{jammer}}$$

Attenuation (dB)
(|A_{max}|)

$$-80$$



Functions

$$\text{sallenkey}(f_0, Q, \text{SNR}, G) := \left[\begin{array}{l} \text{errval} \leftarrow \text{if} \left[\left(G > \frac{1}{8 \cdot Q^2} + 1 \right), \left(G < 1 + \frac{1}{Q} - \frac{1}{2 \cdot Q^2} \right), 1, 0 \right] \\ \text{choice} \leftarrow \text{if} \left[\left(G < \frac{1}{8 \cdot Q^2} + 1, 0, 1 \right) \right] \\ R_1 \leftarrow \text{if} \left[\text{choice}, \left[\frac{\frac{1}{2} - Q^2 \cdot (1 - G)}{(1 - G)^2 \cdot Q^2 + 1} \right], \frac{V_{\text{neq}}^2 - V_n^2}{4 \cdot k \cdot \text{Temp}} \cdot \left[1 - \sqrt{1 - \frac{1}{\left[\frac{1}{2 \cdot Q} - Q \cdot (1 - G) \right]^2}} \right], \frac{V_{\text{neq}}^2 - V_n^2}{2 \cdot 4 \cdot k \cdot \text{Temp}} \right] \\ \text{if} (R_1 > R_{\text{max}}, R_{\text{max}}, R_1) \\ R_2 \leftarrow \text{if} \left[\text{choice}, \left[\frac{V_{\text{neq}}^2 - V_n^2}{4 \cdot k \cdot \text{Temp}} - R_1, R_1 \right] \right] \\ C_2 \leftarrow \text{if} \left[\text{choice}, \left[\frac{1}{[R_1 + R_2 + (1 - G) \cdot R_1] \cdot Q \cdot \omega_0}, \frac{1 + \sqrt{1 - 8 \cdot Q^2 \cdot (G - 1)}}{4 \cdot Q \cdot \omega_0 \cdot R} \right] \right] \\ C_1 \leftarrow \text{if} \left[\text{choice}, \left[\frac{1}{[R_1 + R_2 + (1 - G) \cdot R_1] \cdot Q \cdot \omega_0}, \frac{1}{\omega_0^2 \cdot R^2 \cdot C_2} \right] \right] \\ \left(\begin{array}{l} \text{errval} \\ R_1 \\ \Omega \\ R_2 \end{array} \right) \end{array} \right]$$

$$\left(\begin{array}{c} \frac{\Omega}{F} \\ \frac{C_1}{F} \\ \frac{C_2}{F} \end{array} \right)$$

Example

$f_0 = 613.5 \text{ kHz}$

$Q = 0.707$

$\text{SNR} = 80$

$G = 1$

$x := \text{sallenkey}(f_0, Q, \text{SNR}, G)$

$\text{errval} := x_1$

$R_1 := x_2 \cdot \Omega$

$R_2 := x_3 \cdot \Omega$

$C_1 := x_4 \cdot F$

$C_2 := x_5 \cdot F$

$\text{errval} = 0$

$R_1 = 182.794 \text{ k}\Omega$

$R_2 = 182.794 \text{ k}\Omega$

$C_1 = 3.669 \text{ pF}$

$C_2 = 1.834 \text{ pF}$

Center Frequency

Desired Q

Minimum Signal to Noise Ratio

Gain

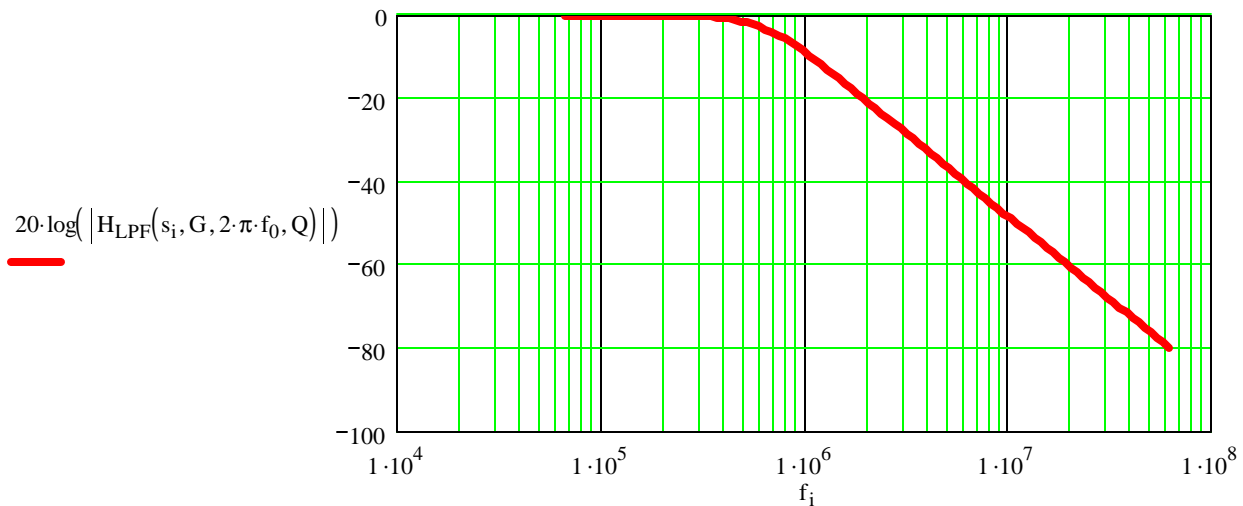
Error? (0=error, 1=no error)

Resistor 1 Value

Resistor 2 Value

Capacitor 1 Value

Capacitor 2 Value

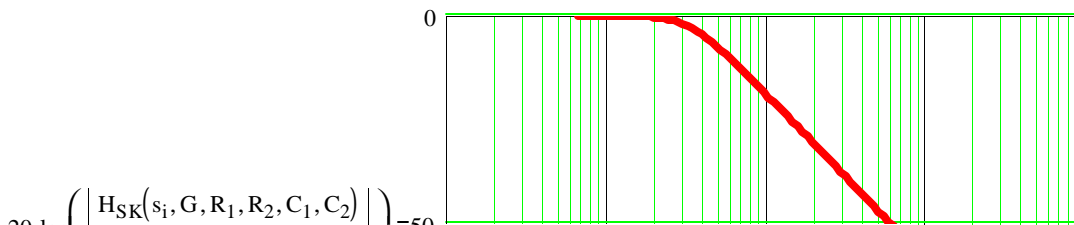


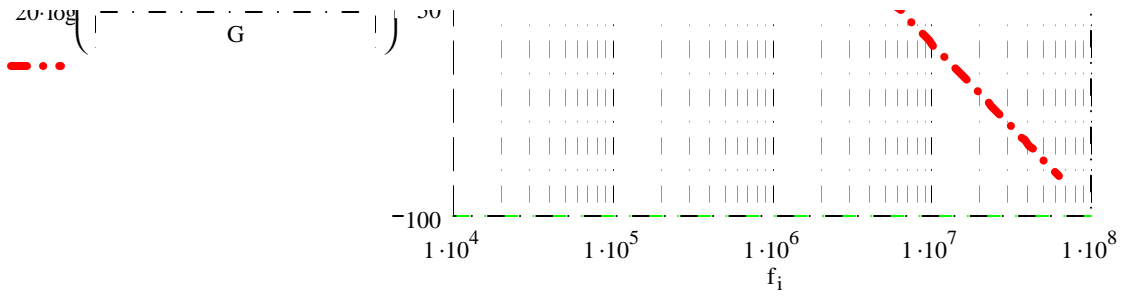
Analysis yields:

$$\omega_0(R_1, R_2, C_1, C_2) := \frac{1}{\sqrt{R_1 \cdot R_2 \cdot C_1 \cdot C_2}} \quad \frac{\omega_0(R_1, R_2, C_1, C_2)}{2 \cdot \pi} = 0.336 \text{ MHz} \quad \omega_0 = \frac{1}{\sqrt{R_1 \cdot R_2 \cdot C_1 \cdot C_2}}$$

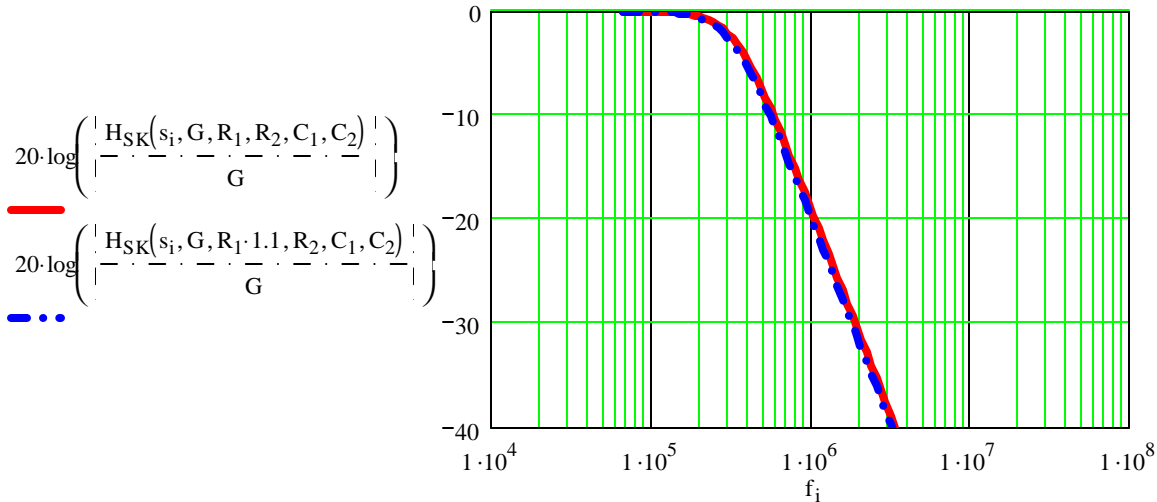
$$Q(G, R_1, R_2, C_1, C_2) := \frac{1}{\frac{1}{R_1 \cdot C_1} + \frac{1}{R_2 \cdot C_1} + \frac{1-G}{R_2 \cdot C_2}} \quad Q(G, R_1, R_2, C_1, C_2) = 0.707 \quad Q = \frac{\sqrt{C_1}}{C_2} \cdot \frac{\sqrt{R_1 \cdot R_2}}{R_1 + R_2} = \frac{1}{C_2 \cdot \omega_0 \cdot (R_1 + R_2)}$$

$$H_{\text{SK}}(s, G, R_1, R_2, C_1, C_2) := H_{\text{LPF}}(s, G, \omega_0(R_1, R_2, C_1, C_2), Q(G, R_1, R_2, C_1, C_2))$$





"small" variations



$$\frac{Q(G, 1.1 \cdot R_1, R_2, C_1, C_2)}{Q(G, R_1, R_2, C_1, C_2)} - 1 = -1.134 \times 10^{-3} \quad \frac{\omega_0(1.1 \cdot R_1, R_2, C_1, C_2)}{\omega_0(R_1, R_2, C_1, C_2)} - 1 = -0.047$$

Sensitivity

Version 1: $S_{QR1}(Q) := Q - \frac{1}{2} \quad S_{QR1}(5) = 4.5$

$$\Delta R_1 := 0.1 \cdot R_1 \quad \Delta Q \text{ by } Q := S_{QR1}(5) \cdot \frac{\Delta R_1}{R_1} \quad \Delta Q \text{ by } Q = 0.45$$

Version 2: $S_{QR1}(Q) := 0$

Note: true only for very small variations ... large (e.g. 10%) changes of R_1 will still change Q .

Conclusions

choose $G=1$ for minimum sensitivity!

Other benefits of $G=1$:

- simplicity, low sensitivity of G ...
- good linearity: voltage across R is zero in passband

Other Problems:

- sensitive to top/bottom plate parasitics
- sensitive to RC time constant variations

Use only in non-critical applications:

- low pole Q (hence restrict order to 4 or so)
- where large variations of cutoff frequency can be accepted (typical variation of untrimmed RC time constants: +/- 30%)

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