

Elliptical Filter Design

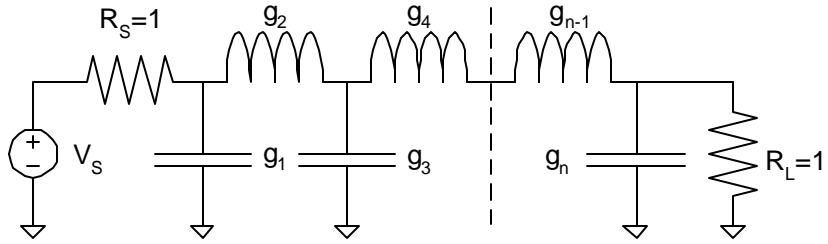


Fig. 1: Lowpass LC filter used for odd-order analysis

- ▢ useful functions and identities
- ▢ Units
- ▢ Constants

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Introduction

$$A(f) = 10 \cdot \log \left[1 + \left(\frac{\text{Ripple}}{10^{10} - 1} \right) \cdot \cos \left(n \cdot \arccos \left(\frac{f}{f_c} \right) \right)^2 \right]$$

Inband Equiripple passband Filter Response
 $f < f_c$

$$A(f) = 10 \cdot \log \left[1 + \left(\frac{\text{Ripple}}{10^{10} - 1} \right) \cdot \cos \left(n \cdot \arccos \left(\frac{f}{f_c} \right) \right)^2 \right]$$

Outband Equiripple passband Filter Response
 $f > f_c$

Inputs

- Atten := 40dB
- Ripple := 1dB
- f_p := 1kHz
- f_s := 2kHz
- R_S := 50ohm
- R_L := 50ohm

- Stop-Band Attenuation
- Maximum Passband Ripple
- Passband corner frequency
- Stopband corner frequency
- Source Impedance
- Load Impedance

Filter Calculations

Order calculation holds ripple constant and chooses n to meet attenuation

$$n := \text{ceil} \left(\frac{\text{acosh} \left(\sqrt{\frac{\frac{\text{Atten}}{10} - 1}{\frac{\text{Ripple}}{10} - 1}} \right)}{\text{acosh} \left(\frac{f_s}{f_p} \right)} \right) \quad n = 5 \quad \text{Required Order of Elliptic Filter}$$

$$\text{Aatfp} := 10 \cdot \log \left[1 + \left(\frac{\text{Ripple}}{10} - 1 \right) \cdot \cos \left(n \cdot \text{acos} \left(\frac{f_p}{f_p} \right) \right)^2 \right] \quad \text{Aatfp} = 1 \text{ dB} \quad \text{Passband Ripple}$$

$$\text{Aatfs} := 10 \cdot \log \left[1 + \left(\frac{\text{Ripple}}{10} - 1 \right) \cdot \cos \left(n \cdot \text{acos} \left(\frac{f_s}{f_p} \right) \right)^2 \right] \quad \text{Aatfs} = 45.306 \text{ dB} \quad \text{Stopband Attenuation}$$

 Images

Element Values for Equal Load and Source Impedances[1]


This method assumes the inductor is the first element

 Create Circuit

$$Z_S = 50 \Omega$$

$$Z_L = 50 \Omega$$

HighPass Filter Calculation

 Create Circuit

$$Z_S = 50 \Omega$$

$$Z_L = 50 \Omega$$

BandPass Filter Calculation

 Create Circuit

$$Z_S = 50 \Omega \quad Z_L = 50 \Omega$$

Functions

$$\begin{aligned}
 \text{lowellip}(\text{Atten}, \text{Ripple}, f_p, f_s, R_S) := & \left(\begin{array}{l} \left(\begin{array}{l} \text{acosh} \left(\frac{\sqrt{\frac{\text{Atten}}{10} \frac{10}{10} - 1}}{\text{Ripple}} \right) \\ \text{acosh} \left(\frac{f_s}{f_p} \right) \end{array} \right) \\ n \leftarrow \text{ceil} \left(\frac{\quad}{\quad} \right) \end{array} \right) \\
 g_1 \leftarrow & \frac{2 \cdot \sin \left(\frac{\pi \cdot 2}{n} \right)}{\sinh \left(\frac{\beta}{2 \cdot n} \right)} \\
 \text{for } k \in & 1..n-1 \\
 g_{k+1} \leftarrow & \text{if } \left[\begin{array}{l} k = n, \text{ if } \left(\text{ceil} \left(\frac{n}{2} \right) > \frac{n}{2} \right), 1, \text{ coth} \left(\frac{\beta}{4} \right)^2 \end{array} \right], \frac{4 \cdot \sin \left[\frac{(2 \cdot k - 1) \cdot \pi}{2 \cdot n} \right] \cdot \sin \left[\frac{(2 \cdot k - 1) \cdot \pi}{2 \cdot n} \right]}{\sinh \left(\frac{\beta}{2 \cdot n} \right)^2 + \sin \left(\frac{k \cdot \pi}{2 \cdot n} \right)^2} \cdot g_k \\
 \text{numL} \leftarrow & \text{if} \left(\text{ceil} \left(\frac{n}{2} \right) > \frac{n}{2}, \frac{n-1}{2}, \frac{n}{2} \right) \\
 \text{numC} \leftarrow & \text{if} \left(\text{ceil} \left(\frac{n}{2} \right) > \frac{n}{2}, \frac{n+1}{2}, \frac{n}{2} \right) \\
 \text{for } i_L \in & 1.. \text{numL} \\
 L_{i_L} \leftarrow & \frac{R_S}{2 \cdot \pi \cdot f_p} \cdot g_{2 \cdot i_L} \\
 \text{for } i_C \in & 1.. \text{numC} \\
 C_{i_C} \leftarrow & \frac{1}{2 \cdot \pi \cdot f_p \cdot R_S} \cdot g_{2 \cdot i_C - 1} \\
 \left(\begin{array}{l} L \\ H \\ C \\ F \end{array} \right)
 \end{aligned}$$

$$\text{highellip}(\text{Atten}, \text{Ripple}, f_p, f_s, R_S) := \left(\begin{array}{l}
n \leftarrow \text{ceil} \left(\frac{\text{acosh} \left(\sqrt{\frac{\frac{\text{Atten}}{10^{10}} - 1}{\text{Ripple}}} \right)}{\text{acosh} \left(\frac{f_s}{f_p} \right)} \right) \\
g_1 \leftarrow \frac{2 \cdot \sin \left(\frac{\pi \cdot 2}{n} \right)}{\sinh \left(\frac{\beta}{2 \cdot n} \right)} \\
\text{for } k \in 1..n-1 \\
g_{k+1} \leftarrow \left[k = n, \text{if} \left(\text{ceil} \left(\frac{n}{2} \right) > \frac{n}{2}, 1, \text{coth} \left(\frac{\beta}{4} \right) \right)^2 \right] \cdot \frac{4 \cdot \sin \left[\frac{(2 \cdot k - 1) \cdot \pi}{2 \cdot n} \right] \cdot \sin \left[\frac{(2 \cdot k - 1) \cdot \pi}{2 \cdot n} \right]}{\sinh \left(\frac{\beta}{2 \cdot n} \right)^2 + \sin \left(\frac{k \cdot \pi}{2 \cdot n} \right)^2} \cdot g_k \\
\text{numL} \leftarrow \text{if} \left(\text{ceil} \left(\frac{n}{2} \right) > \frac{n}{2}, \frac{n+1}{2}, \frac{n}{2} \right) \\
\text{numC} \leftarrow \text{if} \left(\text{ceil} \left(\frac{n}{2} \right) > \frac{n}{2}, \frac{n-1}{2}, \frac{n}{2} \right) \\
\text{for } i_L \in 1.. \text{numL} \\
L_{i_L} \leftarrow \frac{R_S}{2 \cdot \pi \cdot f_p} \cdot \frac{1}{g_{2 \cdot i_L - 1}} \\
\text{for } i_C \in 1.. \text{numC} \\
C_{i_C} \leftarrow \frac{1}{2 \cdot \pi \cdot f_p \cdot R_S} \cdot \frac{1}{g_{2 \cdot i_C}} \\
\left(\begin{array}{l}
L \\
H \\
C \\
F
\end{array} \right)
\end{array} \right)$$

$$\begin{aligned}
\text{bandellip}(\text{Atten}, \text{Ripple}, f_p, f_s, R_S) := & \left(\begin{array}{l} n \leftarrow \text{ceil} \left(\frac{\text{acosh} \left(\sqrt{\frac{\frac{\text{Atten}}{10} \cdot 10 - 1}{\frac{\text{Ripple}}{10} \cdot 10 - 1}} \right)}{\text{acosh} \left(\frac{f_s}{f_p} \right)} \right) \\ \\ g_1 \leftarrow \frac{2 \cdot \sin \left(\frac{\pi \cdot 2}{n} \right)}{\sinh \left(\frac{\beta}{2 \cdot n} \right)} \\ \text{for } k \in 1..n-1 \\ \\ g_{k+1} \leftarrow \text{if } k = n, \text{if } \left(\text{ceil} \left(\frac{n}{2} \right) > \frac{n}{2}, 1, \text{coth} \left(\frac{\beta}{4} \right)^2 \right), \frac{4 \cdot \sin \left[\frac{(2 \cdot k - 1) \cdot \pi}{2 \cdot n} \right] \cdot \sin \left[\frac{(2 \cdot k - 1) \cdot \pi}{2 \cdot n} \right]}{\sinh \left(\frac{\beta}{2 \cdot n} \right)^2 + \sin \left(\frac{k \cdot \pi}{2 \cdot n} \right)^2} \cdot g_k \\ \\ \text{numT} \leftarrow \text{if} \left(\text{ceil} \left(\frac{n}{2} \right) > \frac{n}{2}, \frac{n+1}{2}, \frac{n}{2} \right) \\ \text{numB} \leftarrow \text{if} \left(\text{ceil} \left(\frac{n}{2} \right) > \frac{n}{2}, \frac{n-1}{2}, \frac{n}{2} \right) \\ \text{for } iT \in 1.. \text{numT} \\ \left| \begin{array}{l} L_{2 \cdot iT - 1} \leftarrow \frac{R_S}{2 \cdot \pi \cdot f_p} \cdot g_{2 \cdot iT - 1} \cdot w \\ C_{2 \cdot iT - 1} \leftarrow \frac{1}{2 \cdot \pi \cdot f_p \cdot R_S} \cdot \frac{1}{g_{2 \cdot iT - 1} \cdot w} \end{array} \right. \\ \text{for } iB \in 1.. \text{numB} \\ \left| \begin{array}{l} C_{2 \cdot iB} \leftarrow \frac{1}{2 \cdot \pi \cdot f_p \cdot R_S} \cdot g_{2 \cdot iB} \cdot w \\ L_{2 \cdot iB} \leftarrow \frac{R_S}{2 \cdot \pi \cdot f_p} \cdot \frac{1}{g_{2 \cdot iB} \cdot w} \end{array} \right. \\ \left(\begin{array}{l} L \\ H \\ C \\ F \end{array} \right) \end{array} \right)
\end{aligned}$$

Example

LC := lowellip(Atten, Ripple, f_p , f_s , RS)

L := LC₁ · H

C := LC₂ · F

LC := highellip(Atten, Ripple, f_p , f_s , RS)

L := LC₁ · H

C := LC₂ · F

LC := bandellip(Atten, Ripple, f_p , f_s , RS)

L := LC₁ · H

C := LC₂ · F

$$L^T = \begin{pmatrix} 4.274 \times 10^{-3} & 4.938 \times 10^{-3} \end{pmatrix} H$$
$$C^T = \begin{pmatrix} 2.092 \times 10^{-5} & 3.094 \times 10^{-5} & 1.292 \times 10^{-5} \end{pmatrix} F$$

$$L^T = \begin{pmatrix} 1.211 \times 10^{-3} & 8.187 \times 10^{-4} & 1.961 \times 10^{-3} \end{pmatrix} H$$
$$C^T = \begin{pmatrix} 5.927 \times 10^{-6} & 5.129 \times 10^{-6} \end{pmatrix} F$$

$$L^T = \begin{pmatrix} 0.052 & 0.015 & 0.077 & 0.013 & 0.032 \end{pmatrix} H$$
$$C^T = \begin{pmatrix} 4.844 \times 10^{-7} & 1.709 \times 10^{-6} & 3.275 \times 10^{-7} & 1.975 \times 10^{-7} \end{pmatrix} F$$

References

Microwave Electronic Circuit Technology, by Yoshihiro Konishi, Marcel Dekker Publishing, New York, 1998, Filter Design Equations: pp 199-

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