Elliptical Filter Design

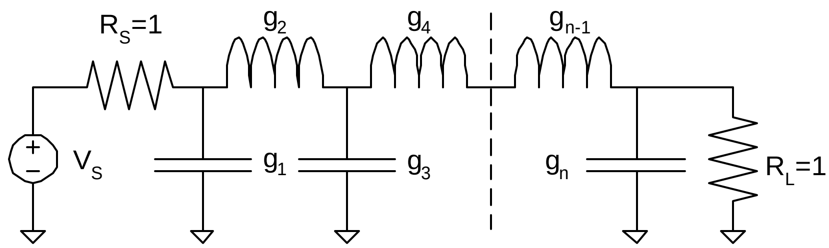

Fig. 1: Lowpass LC filter used for odd-order analysis

▸ useful functions and identities
▸ Units
▸ Constants

Table of Contents

I. Introduction
IV. Inputs
IX. Copyright and Trademark Notice

Introduction

$$A(f) = 10 \cdot \log\left[1 + \left(10^{\frac{Ripple}{10}} - 1\right) \cdot \cos\left(n \cdot \mathrm{acos}\left(\frac{f}{f_c}\right)\right)^2\right]$$

Inband Equiripple passband Filter Response

$f < fc$

$$A(f) = 10 \cdot \log\left[1 + \left(10^{\frac{Ripple}{10}} - 1\right) \cdot \cos\left(n \cdot \mathrm{acos}\left(\frac{f}{f_c}\right)\right)^2\right]$$

Outband Equiripple passband Filter Response

$f > fc$

Inputs

$Atten := 40dB$	Stop-Band Attenuation
$Ripple := 1dB$	Maximum Passband Ripple
$f_p := 1kHz$	Passband corner frequency
$f_s := 2kHz$	Stopband corner frequency
$R_S := 50ohm$	Source Impedance
$R_L := 50ohm$	Load Impedance

Filter Calculations

Order calculation holds ripple constant and chooses n to meet attenuation

$$n := \mathrm{ceil}\left(\frac{\mathrm{acosh}\left(\sqrt{\dfrac{10^{\frac{Atten}{10}}-1}{10^{\frac{Ripple}{10}}-1}}\right)}{\mathrm{acosh}\left(\dfrac{f_s}{f_p}\right)}\right)$$

$n = 5$ Required Order of Elliptic Filter

$$Aatfp := 10\cdot\log\left[1+\left(10^{\frac{Ripple}{10}}-1\right)\cdot\cos\left(n\cdot\mathrm{acos}\left(\frac{f_p}{f_p}\right)\right)^2\right]$$

$Aatfp = 1\,dB$ Passband Ripple

$$Aatfs := 10\cdot\log\left[1+\left(10^{\frac{Ripple}{10}}-1\right)\cdot\cos\left(n\cdot\mathrm{acos}\left(\frac{f_s}{f_p}\right)\right)^2\right]$$

$Aatfs = 45.306\,dB$ Stopband Attenuation

▶ Images

Element Values for Equal Load and Source Impedances[1]

This method assumes the inductor is the first element

▶ Create Circuit

$Z_S = 50\,\Omega$

$Z_L = 50\,\Omega$

HighPass Filter Calculation

▶ Create Circuit

$Z_S = 50\,\Omega$

$Z_L = 50\,\Omega$

BandPass Filter Calculation

▶ Create Circuit

$Z_S = 50\,\Omega$ $Z_L = 50\,\Omega$

Functions

$$
\text{lowellip}(\text{Atten}, \text{Ripple}, f_p, f_s, R_S) :=
\begin{array}{|l}
n \leftarrow \text{ceil}\left(\dfrac{\text{acosh}\left(\sqrt{\dfrac{10^{\frac{\text{Atten}}{10}} - 1}{10^{\frac{\text{Ripple}}{10}} - 1}} \right)}{\text{acosh}\left(\dfrac{f_s}{f_p} \right)} \right) \\
g_1 \leftarrow \dfrac{2 \cdot \sin\left(\dfrac{\pi \cdot 2}{n} \right)}{\sinh\left(\dfrac{\beta}{2 \cdot n} \right)} \\
\text{for } k \in 1..\, n - 1 \\
\quad g_{k+1} \leftarrow \text{if}\left[k = n, \text{if}\left(\text{ceil}\left(\dfrac{n}{2} \right) > \dfrac{n}{2}, 1, \coth\left(\dfrac{\beta}{4} \right)^2 \right), \dfrac{4 \cdot \sin\left[\dfrac{(2 \cdot k - 1) \cdot \pi}{2 \cdot n} \right] \cdot \sin\left[\dfrac{(2 \cdot k - 1) \cdot \pi}{2 \cdot n} \right.}{\sinh\left(\dfrac{\beta}{2 \cdot n} \right)^2 + \sin\left(\dfrac{k \cdot \pi}{2 \cdot n} \right)^2 \cdot g_k} \right. \\
\text{numL} \leftarrow \text{if}\left(\text{ceil}\left(\dfrac{n}{2} \right) > \dfrac{n}{2}, \dfrac{n - 1}{2}, \dfrac{n}{2} \right) \\
\text{numC} \leftarrow \text{if}\left(\text{ceil}\left(\dfrac{n}{2} \right) > \dfrac{n}{2}, \dfrac{n + 1}{2}, \dfrac{n}{2} \right) \\
\text{for } iL \in 1..\, \text{numL} \\
\quad L_{iL} \leftarrow \dfrac{R_S}{2 \cdot \pi \cdot f_p} \cdot g_{2 \cdot iL} \\
\text{for } iC \in 1..\, \text{numC} \\
\quad C_{iC} \leftarrow \dfrac{1}{2 \cdot \pi \cdot f_p \cdot R_S} \cdot g_{2 \cdot iC - 1} \\
\begin{pmatrix} \dfrac{L}{H} \\ \dfrac{C}{F} \end{pmatrix}
\end{array}
$$

$$\mathrm{highellip}(\mathrm{Atten},\mathrm{Ripple},f_p,f_s,R_S) := $$

$$n \leftarrow \mathrm{ceil}\left(\frac{\mathrm{acosh}\left(\sqrt{\dfrac{10^{\frac{\mathrm{Atten}}{10}}-1}{10^{\frac{\mathrm{Ripple}}{10}}-1}}\right)}{\mathrm{acosh}\left(\dfrac{f_s}{f_p}\right)}\right)$$

$$g_1 \leftarrow \frac{2\cdot\sin\left(\dfrac{\pi\cdot 2}{n}\right)}{\sinh\left(\dfrac{\beta}{2\cdot n}\right)}$$

$$\text{for } k \in 1..n-1$$

$$g_{k+1} \leftarrow \mathrm{if}\left[k = n, \mathrm{if}\left(\mathrm{ceil}\left(\frac{n}{2}\right) > \frac{n}{2}, 1, \coth\left(\frac{\beta}{4}\right)^2\right), \frac{4\cdot\sin\left[\dfrac{(2\cdot k-1)\cdot\pi}{2\cdot n}\right]\cdot\sin\left[\dfrac{(2\cdot k-1)\cdot \ldots}{2\cdot n}\right.}{\sinh\left(\dfrac{\beta}{2\cdot n}\right)^2 + \sin\left(\dfrac{k\cdot\pi}{2\cdot n}\right)^2\cdot g_k}\right.$$

$$\mathrm{numL} \leftarrow \mathrm{if}\left(\mathrm{ceil}\left(\frac{n}{2}\right) > \frac{n}{2}, \frac{n+1}{2}, \frac{n}{2}\right)$$

$$\mathrm{numC} \leftarrow \mathrm{if}\left(\mathrm{ceil}\left(\frac{n}{2}\right) > \frac{n}{2}, \frac{n-1}{2}, \frac{n}{2}\right)$$

$$\text{for } iL \in 1..\mathrm{numL}$$

$$L_{iL} \leftarrow \frac{R_S}{2\cdot\pi\cdot f_p}\cdot\frac{1}{g_{2\cdot iL-1}}$$

$$\text{for } iC \in 1..\mathrm{numC}$$

$$C_{iC} \leftarrow \frac{1}{2\cdot\pi\cdot f_p\cdot R_S}\cdot\frac{1}{g_{2\cdot iC}}$$

$$\begin{pmatrix} \dfrac{L}{H} \\ \dfrac{C}{F} \end{pmatrix}$$

$$
\begin{aligned}
\mathrm{bandellip}(\mathrm{Atten}, \mathrm{Ripple}, f_p, f_s, R_S) := \quad & n \leftarrow \mathrm{ceil}\left(\frac{\mathrm{acosh}\left(\sqrt{\dfrac{10^{\frac{\mathrm{Atten}}{10}} - 1}{10^{\frac{\mathrm{Ripple}}{10}} - 1}} \right)}{\mathrm{acosh}\left(\dfrac{f_s}{f_p} \right)} \right) \\
& g_1 \leftarrow \frac{2 \cdot \sin\left(\dfrac{\pi \cdot 2}{n} \right)}{\sinh\left(\dfrac{\beta}{2 \cdot n} \right)} \\
& \text{for } k \in 1 .. n - 1 \\
& \quad g_{k+1} \leftarrow \mathrm{if}\left[k = n, \mathrm{if}\left(\mathrm{ceil}\left(\frac{n}{2} \right) > \frac{n}{2}, 1, \coth\left(\frac{\beta}{4} \right)^2 \right), \frac{4 \cdot \sin\left[\dfrac{(2 \cdot k - 1) \cdot \pi}{2 \cdot n} \right] \cdot \sin\left[\dfrac{(2 \cdot k - 1) \cdot}{2 \cdot n} \right.}{\sinh\left(\dfrac{\beta}{2 \cdot n} \right)^2 + \sin\left(\dfrac{k \cdot \pi}{2 \cdot n} \right)^2 \cdot g_k} \right. \\
& \mathrm{numT} \leftarrow \mathrm{if}\left(\mathrm{ceil}\left(\frac{n}{2} \right) > \frac{n}{2}, \frac{n + 1}{2}, \frac{n}{2} \right) \\
& \mathrm{numB} \leftarrow \mathrm{if}\left(\mathrm{ceil}\left(\frac{n}{2} \right) > \frac{n}{2}, \frac{n - 1}{2}, \frac{n}{2} \right) \\
& \text{for } iT \in 1 .. \mathrm{numT} \\
& \quad L_{2 \cdot iT - 1} \leftarrow \frac{R_S}{2 \cdot \pi \cdot f_p} \cdot g_{2 \cdot iT - 1} \cdot w \\
& \quad C_{2 \cdot iT - 1} \leftarrow \frac{1}{2 \cdot \pi \cdot f_p \cdot R_S} \cdot \frac{1}{g_{2 \cdot iT - 1} \cdot w} \\
& \text{for } iB \in 1 .. \mathrm{numB} \\
& \quad C_{2 \cdot iB} \leftarrow \frac{1}{2 \cdot \pi \cdot f_p \cdot R_S} \cdot g_{2 \cdot iB} \cdot w \\
& \quad L_{2 \cdot iB} \leftarrow \frac{R_S}{2 \cdot \pi \cdot f_p} \cdot \frac{1}{g_{2 \cdot iB} \cdot w} \\
& \begin{pmatrix} \dfrac{L}{H} \\ \dfrac{C}{F} \end{pmatrix}
\end{aligned}
$$

Example

$LC := \text{lowellip}(\text{Atten}, \text{Ripple}, f_p, f_s, R_S)$

$L := LC_1 \cdot H$ $\qquad L^T = (4.274 \times 10^{-3} \quad 4.938 \times 10^{-3})\, H$

$C := LC_2 \cdot F$ $\qquad C^T = (2.092 \times 10^{-5} \quad 3.094 \times 10^{-5} \quad 1.292 \times 10^{-5})\, F$

$LC := \text{highellip}(\text{Atten}, \text{Ripple}, f_p, f_s, R_S)$

$L := LC_1 \cdot H$ $\qquad L^T = (1.211 \times 10^{-3} \quad 8.187 \times 10^{-4} \quad 1.961 \times 10^{-3})\, H$

$C := LC_2 \cdot F$ $\qquad C^T = (5.927 \times 10^{-6} \quad 5.129 \times 10^{-6})\, F$

$LC := \text{bandellip}(\text{Atten}, \text{Ripple}, f_p, f_s, R_S)$

$L := LC_1 \cdot H$ $\qquad L^T = (0.052 \quad 0.015 \quad 0.077 \quad 0.013 \quad 0.032)\, H$

$C := LC_2 \cdot F$ $\qquad C^T = (4.844 \times 10^{-7} \quad 1.709 \times 10^{-6} \quad 3.275 \times 10^{-7} \quad 1.975 \times 10^{-}$

References

Microwave Electronic Circuit Technology, by Yoshihiro Konishi, Marcel Dekker Publishing, New York, 1998, Filter Design Equations: pp 199-

Copyright and Trademark Notice