



Chebyshev LC Filter Design

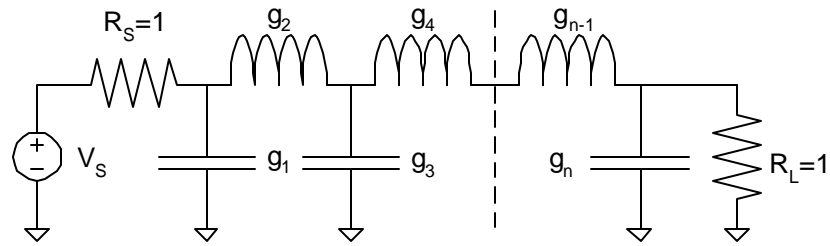


Fig. 1: LC filter used for odd-order analysis

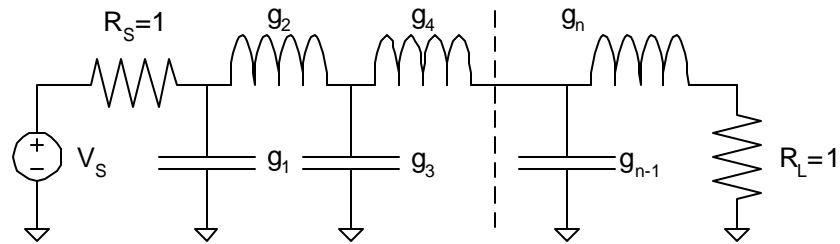


Fig. 2: Filter used for even order analysis

- ▢ useful functions and identities
- ▢ Units
- ▢ Constants

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Inputs

Atten := 70dB
 Ripple := 1dB
 $f_p := \frac{10\text{kHz}}{2 \cdot \pi}$
 $f_s := 10\text{kHz}$
 $R_S := 100\Omega$
 $R_L := 100\Omega$

Stop-band attenuation
 Maximum passband ripple
 Passband corner frequency
 Stopband corner frequency
 Source impedance
 Load impedance

Order Estimation

$$\varepsilon := \sqrt[10]{\frac{\text{Ripple}}{10} - 1} \quad \varepsilon = 0.509 \quad \text{Ripple}$$

$$n := \text{ceil} \left(\frac{\text{acosh} \left(\varepsilon^{-1} \cdot \sqrt[10]{\frac{\text{Atten}}{10} - 1} \right)}{\text{acosh} \left(\frac{f_s}{f_p} \right)} \right) \quad n = 4 \quad \text{Required Order of Chebyshev Filter}$$

$$C(n, \omega) := \text{if}(|\omega| < 1, \cos(n \cdot \arccos(\omega)), \cosh(n \cdot \text{acosh}(\omega))) \quad \text{Chebyshev Coefficients}$$

Check to see if order estimation is correct.

$$\text{Aatfp} := 10 \cdot \log \left(1 + \varepsilon^2 \cdot C \left(n, \frac{f_p}{f_p} \right)^2 \right) \quad \text{Aatfp} = 1 \text{ dB} \quad \text{Passband Ripple}$$

$$\text{Aatfs} := 10 \cdot \log \left(1 + \varepsilon^2 \cdot C \left(n, \frac{f_s}{f_p} \right)^2 \right) \quad \text{Aatfs} = 75.826 \text{ dB} \quad \text{Stopband Attenuation}$$

Poles and Zeros

$$k := 1..n$$

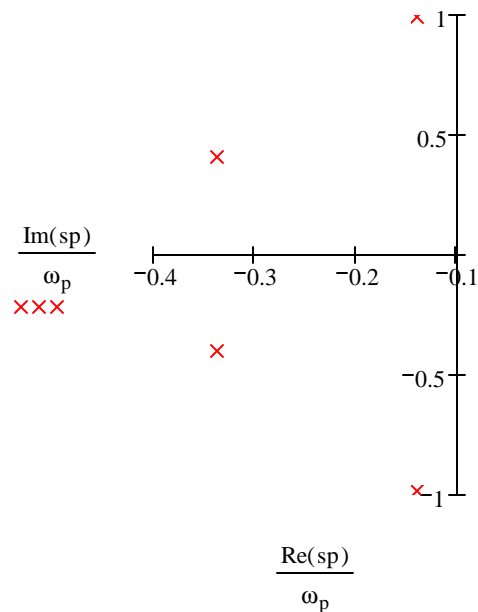
$$\omega_p := 2 \cdot \pi \cdot f_p$$

Index Vector for poles

$$sp_k := -\omega_p \cdot \sinh \left(\frac{1}{n} \cdot \text{asinh} \left(\frac{1}{\varepsilon} \right) \right) \cdot \sin \left[\frac{(2 \cdot k - 1) \cdot \pi}{2 \cdot n} \right] \dots$$

$$+ j \cdot \omega_p \cdot \cosh \left(\frac{1}{n} \cdot \text{asinh} \left(\frac{1}{\varepsilon} \right) \right) \cdot \cos \left[\frac{(2 \cdot k - 1) \cdot \pi}{2 \cdot n} \right]$$

$$sp = \begin{pmatrix} -1.395 + 9.834i \\ -3.369 + 4.073i \\ -3.369 - 4.073i \\ -1.395 - 9.834i \end{pmatrix} \text{ kHz} \quad \text{Poles}$$



Quadratic Sections

$$\frac{1}{\frac{s^2}{\text{re}^2 + \text{im}^2} + \left(\frac{2 \cdot \text{re}}{\text{re}^2 + \text{im}^2}\right)s + 1} = \frac{1}{\left(\frac{s}{\omega}\right)^2 + \frac{1}{Q} \cdot \frac{s}{\omega} + 1} = \frac{1}{\left[\left(\frac{s}{\omega}\right)^2 + 2 \cdot \zeta \cdot \frac{s}{\omega} + 1\right]}$$

$$m := 1.. \text{floor}\left(\frac{n}{2}\right)$$

$$\omega_m := \sqrt{\text{Re}(\text{sp}_m)^2 + \text{Im}(\text{sp}_m)^2}$$

$$Q_m := \frac{-\omega_m}{2 \cdot \text{Re}(\text{sp}_m)}$$

$$\zeta_m := \frac{-\text{Re}(\text{sp}_m)}{\omega_m}$$

$$\omega_{\text{ceil}\left(\frac{n}{2}\right)} := \text{if}\left(\text{ceil}\left(\frac{n}{2}\right) > \frac{n}{2}, -\text{sp}_{\text{ceil}\left(\frac{n}{2}\right)}, 0 \frac{\text{rad}}{\text{sec}}\right)$$

$$\frac{\omega}{2 \cdot \pi} = \begin{pmatrix} 1.581 \\ 0.841 \end{pmatrix} \text{kHz}$$

Quadratic center frequency
(all on unit circle for butterworth)

$$Q = \begin{pmatrix} 3.559 \\ 0.785 \end{pmatrix}$$

Quadratic Q

$$\zeta = \begin{pmatrix} 0.14 \\ 0.637 \end{pmatrix}$$

Quadratic Damping Factor

$$\frac{\omega}{2 \cdot \pi} = \begin{pmatrix} 1.581 \\ 0 \end{pmatrix} \text{kHz}$$

First Order Section

Transfer Function

$$M(s) := \frac{1}{\prod_{k=1}^n \left(1 + \frac{s}{\text{sp}_k}\right)}$$

Transfer Function

$$\text{gd}(f) := \frac{d}{df} \arg \left[\frac{1}{\prod_{k=1}^n \left(1 + \frac{s}{\text{sp}_k}\right)} \right]$$

Group Delay Function

$$\text{phase}(p) := \begin{array}{l} \longrightarrow \\ p \leftarrow \arg(p) \\ \text{return } p \text{ if } \text{last}(p) < 2 \\ \text{wrap} \leftarrow 0 \\ \text{for } i \in 2.. \text{last}(p) \\ \left| \begin{array}{l} \text{wrap} \leftarrow \text{wrap} - 2 \cdot \pi \text{ if } (p_i - p_{i-1} + \text{wrap}) > 3 \\ \text{wrap} \leftarrow \text{wrap} + 2 \cdot \pi \text{ if } (p_i - p_{i-1} + \text{wrap}) < -3 \\ p_i \leftarrow p_i + \text{wrap} \end{array} \right. \\ p \end{array}$$

Phase Function

$$M_2(f) := \frac{1}{\sqrt{1 + \varepsilon^2 \cdot C\left(n, \frac{f}{f_p}\right)^2}}$$

Transfer Function (Eq. 2.10b)

Plotting

num := 300

i := 1.. num

$f_{start} := \frac{f_p}{50}$

$f_{stop} := f_s \cdot 2$

$f_i := f_{start} + \frac{i-1}{num-1} \cdot (f_{stop} - f_{start})$

$s_i := j \cdot (2 \cdot \pi \cdot f_i)$

ang := phase($\overrightarrow{M(s)}$)

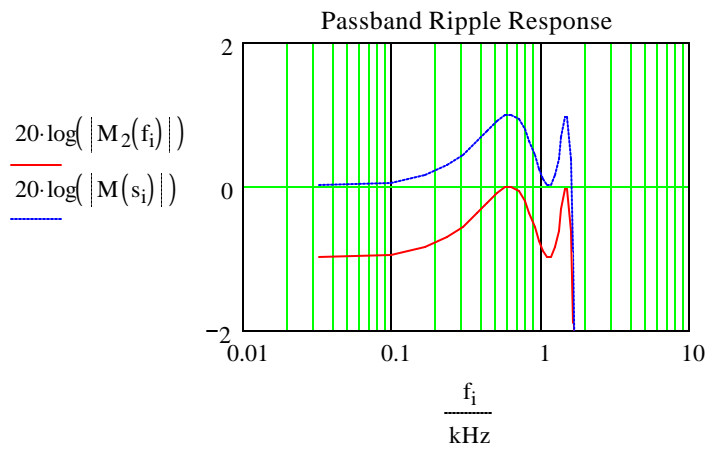
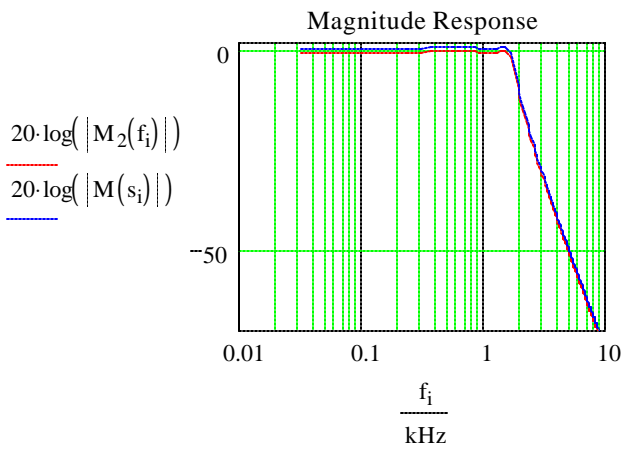
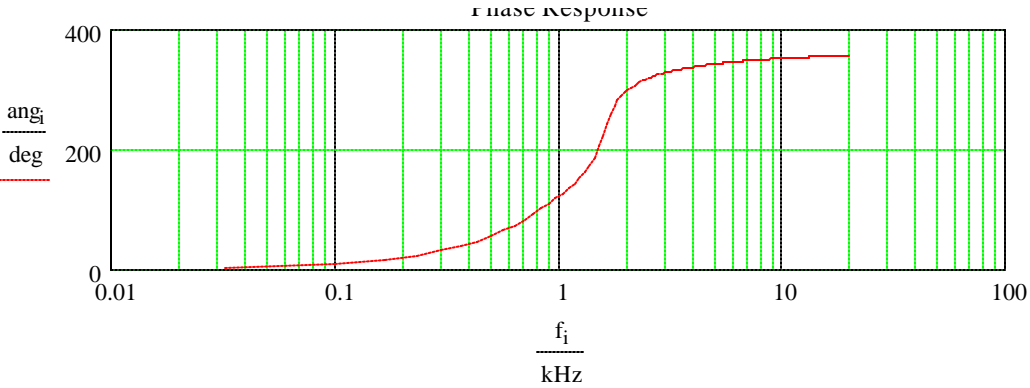
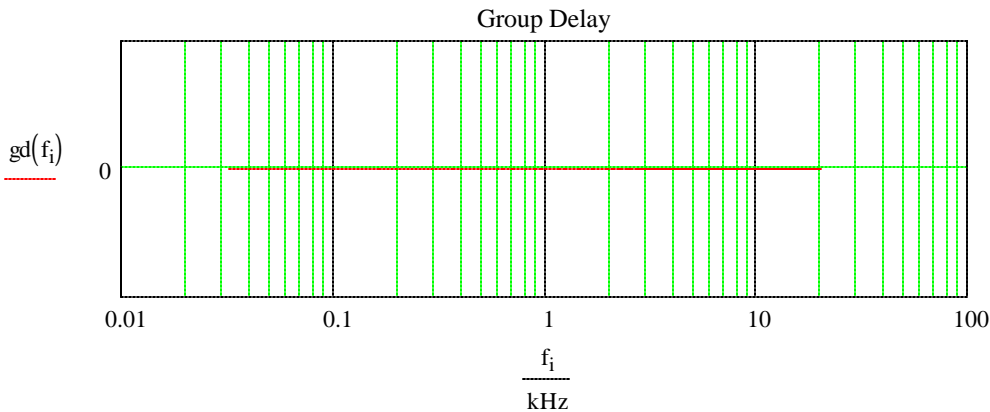
Number of Points for plotting
Frequency Index Vector

Starting Frequency for Plotting

Ending Frequency for Plotting

Frequency Vector


Phase Response



 Images

Element Values for Different Load and Source Impedances[1]

This method assumes the inductor is the first element

 Create Circuit

Element Values for Different Load and Source Impedances[1]

This method assumes the capacitor is the first element. This is true is $\text{Re}(Z_S) > \text{Re}(Z_L)$

 Create Circuit

Functions

$$\begin{aligned}
 \text{cheby}(\text{Atten}, \text{Ripple}, f_p, f_s, R_S, R_L) &:= \text{ceil} \left(\frac{\text{acosh} \left(\varepsilon^{-1} \cdot \sqrt{10^{\frac{\text{Atten}}{10}} - 1} \right)}{\text{acosh} \left(\frac{f_s}{f_p} \right)} \right) \\
 \varepsilon &\leftarrow \sqrt{10^{\frac{\text{Ripple}}{10}} - 1} \\
 K_n &\leftarrow \text{if} \left[\text{ceil} \left(\frac{n}{2} \right) > \frac{n}{2}, 1 - \left(\frac{R_L - R_S}{R_L + R_S} \right)^2, (1 + \varepsilon^2) \cdot \left[1 - \left(\frac{R_L - R_S}{R_L + R_S} \right)^2 \right] \right] \\
 \varepsilon &\leftarrow \text{if} \left[K_n > 1, \frac{1}{\sqrt{\left(\frac{R_L + R_S}{R_L - R_S + .0000001\Omega} \right)^2 - 1}}, \varepsilon \right] \\
 K_n &\leftarrow \text{if} (K_n > 1, 1, K_n) \\
 a &\leftarrow \frac{1}{n} \cdot \text{asinh} \left(\frac{1}{\varepsilon} \right) \\
 ah &\leftarrow \frac{1}{n} \cdot \text{asinh} \left(\frac{\sqrt{1 - K_n}}{\varepsilon} \right) \\
 C_1 &\leftarrow \frac{2 \cdot \sin(\gamma(1))}{\omega_p \cdot R_S \cdot (\sinh(a) - \sinh(ah))} \\
 L_1 &\leftarrow 0H \\
 \text{for } m \in 2..n & \\
 \left[\begin{array}{l} L_m \leftarrow \text{if} \left[\text{ceil} \left(\frac{m}{2} \right) > \frac{m}{2}, 0H, \frac{1}{C_{m-1}} \cdot \frac{1}{\omega_p^2} \cdot \frac{4 \cdot \sin(\gamma(2 \cdot m - 1))}{\sinh(a)^2 + \sinh(ah)^2 + \sin[\gamma[2 \cdot (m - 1)]]} \right. \\ C_m \leftarrow \text{if} \left[\text{ceil} \left(\frac{m}{2} \right) > \frac{m}{2}, \frac{1}{L_{m-1}} \cdot \frac{1}{\omega_p^2} \cdot \frac{4 \cdot \sin(\gamma(2 \cdot m - 3)) \cdot s}{\sinh(a)^2 + \sinh(ah)^2 + \sin[\gamma[2 \cdot (m - 1)]]} \right]^2 \end{array} \right. \\
 \text{augment} \left(\frac{L}{H}, \frac{C}{F} \right) &
 \end{aligned}$$

Example

Atten := 70dB

Ripple := 1dB

$$f_p := \frac{10\text{kHz}}{2 \cdot \pi}$$

$f_s := 10\text{kHz}$

$R_S := 100\Omega$

$R_L := 100\Omega$

LC := cheby(Atten, Ripple, f_p , f_s , R_S , R_L)

$L := LC^{(1)} \cdot H$

$C := LC^{(2)} \cdot F$

Stop-Band Attenuation
Maximum Passband Ripple

Passband corner frequency

Stopband corner frequency

Source Impedance

Load Impedance

$L^T = (0 \ 146.945 \ 0 \ 60.868) \mu\text{H}$

$C^T = (6.087 \ 0 \ 14.694 \ 0) \text{nF}$

References

[1] *Microwave Electronic Circuit Technology*, by Yoshihiro Konishi, Marcel Dekker Publishing, New York, 1998, Filter Design Equations: pp 199-

[2] *Passive and Active Filters, Theory and Implementations*, by Wai-Kai Chen, John Wiley & Sons, New York, 1986, pp. 177-184

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