Chebyshev LC Filter Design

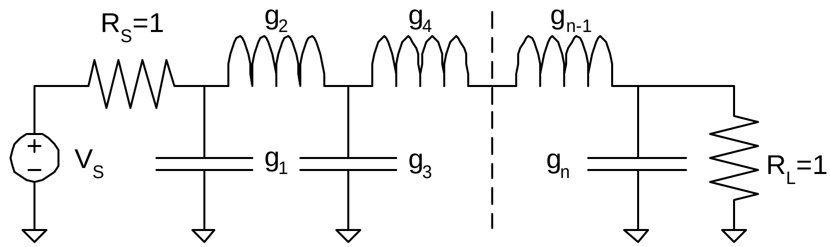

Fig. 1: LC filter used for odd-order analysis

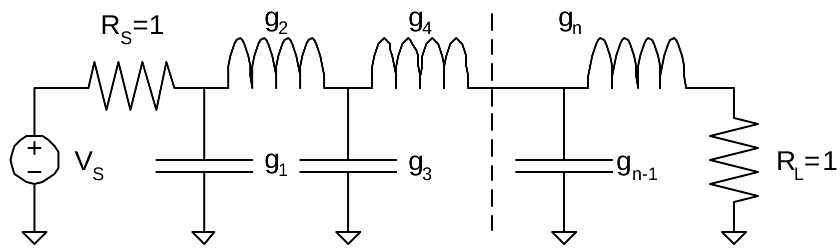

Fig. 2: Filter used for even order analysis

▸ useful functions and identities
▸ Units
▸ Constants

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Inputs

$Atten := 70dB$ Stop-band attenuation

$Ripple := 1dB$ Maximum passband ripple

$f_p := \frac{10kHz}{2\cdot\pi}$ Passband corner frequency

$f_s := 10kHz$ Stopband corner frequency

$R_S := 100\Omega$ Source impedance

$R_L := 100\Omega$ Load impedance

Order Estimation

$$\varepsilon := \sqrt{10^{\frac{Ripple}{10}} - 1} \qquad \varepsilon = 0.509$$ Ripple

$$n := \mathrm{ceil}\left(\frac{\mathrm{acosh}\left(\varepsilon^{-1}\cdot\sqrt{10^{\frac{Atten}{10}} - 1}\right)}{\mathrm{acosh}\left(\frac{f_s}{f_p}\right)}\right) \qquad n = 4$$ Required Order of Chebyshev Filter

$$C(n,\omega) := \mathrm{if}(|\omega| < 1, \cos(n\cdot \mathrm{acos}(\omega)), \cosh(n\cdot \mathrm{acosh}(\omega)))$$ Chebyshev Coefficients

Check to see if order estimation is correct.

$$Aatfp := 10\cdot\log\left(1 + \varepsilon^2\cdot C\left(n, \frac{f_p}{f_p}\right)^2\right) \qquad Aatfp = 1\,dB$$ Passband Ripple

$$Aatfs := 10\cdot\log\left(1 + \varepsilon^2\cdot C\left(n, \frac{f_s}{f_p}\right)^2\right) \qquad Aatfs = 75.826\,dB$$ Stopband Attenuation

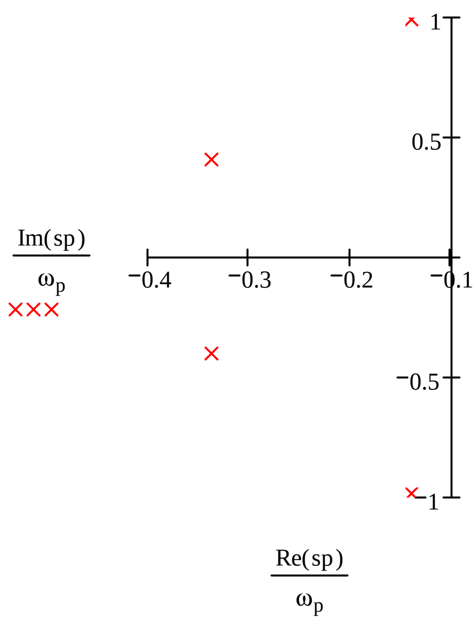
Poles and Zeros

$k := 1..\, n$ Index Vector for poles

$\omega_p := 2\cdot\pi\cdot f_p$

$$sp_k := -\omega_p\cdot\sinh\left(\frac{1}{n}\cdot \mathrm{asinh}\left(\frac{1}{\varepsilon}\right)\right)\cdot\sin\left[\frac{(2\cdot k - 1)\cdot\pi}{2\cdot n}\right] \ldots$$
$$+ j\cdot\omega_p\cdot\cosh\left(\frac{1}{n}\cdot \mathrm{asinh}\left(\frac{1}{\varepsilon}\right)\right)\cdot\cos\left[\frac{(2\cdot k - 1)\cdot\pi}{2\cdot n}\right]$$

$$sp = \begin{pmatrix} -1.395 + 9.834i \\ -3.369 + 4.073i \\ -3.369 - 4.073i \\ -1.395 - 9.834i \end{pmatrix} kHz$$ Poles



Quadratic Sections

$$\frac{1}{\frac{s^2}{re^2+im^2}+\left(\frac{2\cdot re}{re^2+im^2}\right)\cdot s+1} = \frac{1}{\left(\frac{s}{\omega}\right)^2+\frac{1}{Q}\cdot\frac{s}{\omega}+1} = \frac{1}{\left[\left(\frac{s}{\omega}\right)^2+2\cdot\zeta\cdot\frac{s}{\omega}+1\right]}$$

$$m := 1..\,\mathrm{floor}\left(\frac{n}{2}\right)$$

$$\omega_m := \sqrt{\mathrm{Re}\left(sp_m\right)^2+\mathrm{Im}\left(sp_m\right)^2} \qquad \frac{\omega}{2\cdot\pi} = \begin{pmatrix}1.581\\0.841\end{pmatrix}\mathrm{kHz}$$

Quadratic center frequency (all on unit circle for butterworth)

$$Q_m := \frac{-\omega_m}{2\cdot\mathrm{Re}\left(sp_m\right)} \qquad Q = \begin{pmatrix}3.559\\0.785\end{pmatrix}$$

Quadratic Q

$$\zeta_m := \frac{-\mathrm{Re}\left(sp_m\right)}{\omega_m} \qquad \zeta = \begin{pmatrix}0.14\\0.637\end{pmatrix}$$

Quadratic Damping Factor

$$\omega_{\mathrm{ceil}\left(\frac{n}{2}\right)} := \mathrm{if}\left(\mathrm{ceil}\left(\frac{n}{2}\right)>\frac{n}{2},-sp_{\mathrm{ceil}\left(\frac{n}{2}\right)},0\frac{rad}{sec}\right) \qquad \frac{\omega}{2\cdot\pi} = \begin{pmatrix}1.581\\0\end{pmatrix}\mathrm{kHz}$$

First Order Section

Transfer Function

$$M(s) := \frac{1}{\prod_{k=1}^{n}\left(1+\frac{s}{sp_k}\right)}$$

Transfer Function

$$gd(f) := \frac{d}{df}\arg\left[\frac{1}{\prod_{k=1}^{n}\left(1+\frac{s}{sp_k}\right)}\right]$$

Group Delay Function

Phase Function

```
phase(p) := | p ← arg(p)   (vectorized)
            | return p  if  last(p) < 2
            | wrap ← 0
            | for  i ∈ 2.. last(p)
            |    | wrap ← wrap − 2·π  if  (p_i − p_{i−1} + wrap) > 3
            |    | wrap ← wrap + 2·π  if  (p_i − p_{i−1} + wrap) < −3
            |    | p_i ← p_i + wrap
            | p
```

$$M_2(f) := \frac{1}{\sqrt{1+\varepsilon^2\cdot C\left(n,\frac{f}{f_p}\right)^2}}$$

Transfer Function (Eq. 2.10b)

Plotting

$num := 300$ — Number of Points for plotting

$i := 1..num$ — Frequency Index Vector

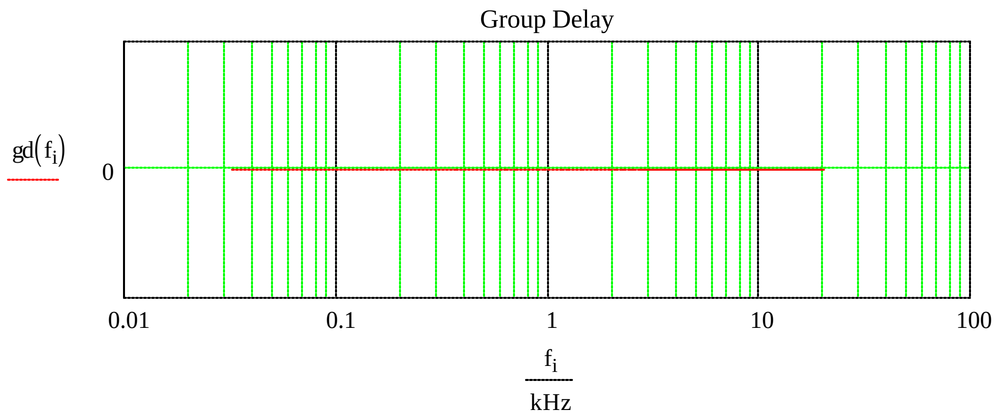
$f_{start} := \frac{f_p}{50}$ — Starting Frequency for Plotting

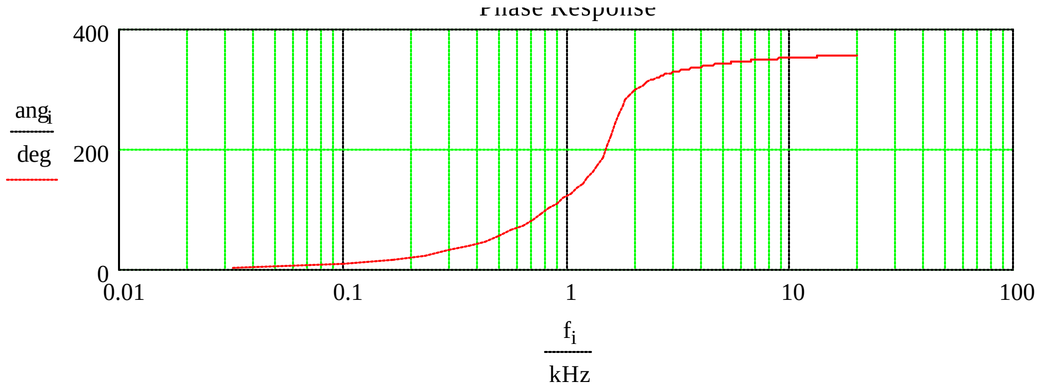
$f_{stop} := f_s \cdot 2$ — Ending Frequency for Plotting

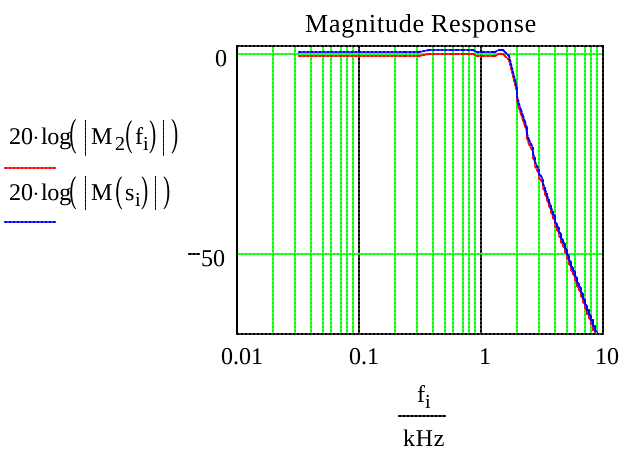
$f_i := f_{start} + \frac{i-1}{num-1} \cdot (f_{stop} - f_{start})$ — Frequency Vector

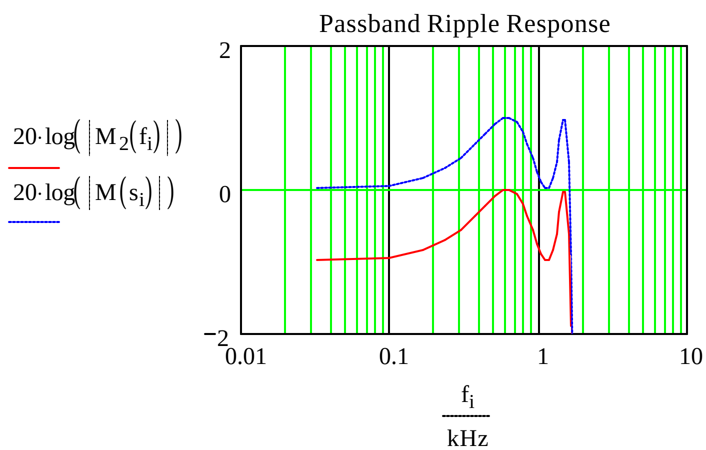
$s_i := j \cdot (2 \cdot \pi \cdot f_i)$

$ang := phase\left(\overrightarrow{M(s)}\right)$ — Phase Response









▸ Images

Element Values for Different Load and Source Impedances[1]

This method assumes the inductor is the first element

▸ Create Circuit

Element Values for Different Load and Source Impedances[1]

This method assumes the capacitor is the first element. This is true is $Re(Z_S)>Re(Z_L)$

▸ Create Circuit

Functions

$$
\mathrm{cheby}(Atten, Ripple, f_p, f_s, R_S, R_L) :=
\left|
\begin{array}{l}
\mathrm{ceil}\left(\dfrac{\mathrm{acosh}\left(\varepsilon^{-1}\cdot\sqrt{10^{\frac{Atten}{10}}-1}\right)}{\mathrm{acosh}\left(\dfrac{f_s}{f_p}\right)}\right) \\
\varepsilon \leftarrow \sqrt{10^{\frac{Ripple}{10}}-1} \\
K_n \leftarrow \mathrm{if}\left[\mathrm{ceil}\left(\dfrac{n}{2}\right) > \dfrac{n}{2}, 1-\left(\dfrac{R_L-R_S}{R_L+R_S}\right)^2, \left(1+\varepsilon^2\right)\cdot\left[1-\left(\dfrac{R_L-R_S}{R_L+R_S}\right)^2\right]\right] \\
\varepsilon \leftarrow \mathrm{if}\left[K_n > 1, \dfrac{1}{\sqrt{\left(\dfrac{R_L+R_S}{R_L-R_S+.0000001\Omega}\right)^2-1}}, \varepsilon\right] \\
K_n \leftarrow \mathrm{if}\left(K_n > 1, 1, K_n\right) \\
a \leftarrow \dfrac{1}{n}\cdot\mathrm{asinh}\left(\dfrac{1}{\varepsilon}\right) \\
ah \leftarrow \dfrac{1}{n}\cdot\mathrm{asinh}\left(\dfrac{\sqrt{1-K_n}}{\varepsilon}\right) \\
C_1 \leftarrow \dfrac{2\cdot\sin(\gamma(1))}{\omega_p\cdot R_S\cdot(\sinh(a)-\sinh(ah))} \\
L_1 \leftarrow 0H \\
\mathrm{for}\ \ m \in 2..n \\
\quad \left|
\begin{array}{l}
L_m \leftarrow \mathrm{if}\left[\mathrm{ceil}\left(\dfrac{m}{2}\right) > \dfrac{m}{2}, 0H, \dfrac{1}{C_{m-1}}\cdot\dfrac{1}{{\omega_p}^2}\cdot\dfrac{4\cdot\sin(\gamma(2\cdot m - \ldots}{\sinh(a)^2+\sinh(ah)^2+\sin[\gamma[2\cdot(m-1)\ldots} \right. \\
C_m \leftarrow \mathrm{if}\left[\mathrm{ceil}\left(\dfrac{m}{2}\right) > \dfrac{m}{2}, \dfrac{1}{L_{m-1}}\cdot\dfrac{1}{{\omega_p}^2}\cdot\dfrac{4\cdot\sin(\gamma(2\cdot m-3))\cdot s\ldots}{\sinh(a)^2+\sinh(ah)^2+\sin[\gamma[2\cdot(m-1)]]^2} \right.
\end{array}
\right. \\
\mathrm{augment}\left(\dfrac{L}{H}, \dfrac{C}{F}\right)
\end{array}
\right.
$$

Example

$Atten := 70dB$ Stop-Band Attenuation

$Ripple := 1dB$ Maximum Passband Ripple

$f_p := \frac{10kHz}{2 \cdot \pi}$ Passband corner frequency

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$R_S := 100\Omega$ Source Impedance

$R_L := 100\Omega$ Load Impedance

$LC := cheby(Atten, Ripple, f_p, f_s, R_S, R_L)$

$L := LC^{\langle 1 \rangle} \cdot H$ $L^T = (0 \quad 146.945 \quad 0 \quad 60.868)\,\mu H$

$C := LC^{\langle 2 \rangle} \cdot F$ $C^T = (6.087 \quad 0 \quad 14.694 \quad 0)\,nF$

References

[1]Microwave Electronic Circuit Technology, by Yoshihiro Konishi, Marcel Dekker Publishing, New York, 1998, Filter Design Equations: pp 199-

[2]Passive and Active Filters, Theory and Implementations, by Wai-Kai Chen, John Wiley & Sons, New York, 1986, pp. 177-184

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